Analysis of the vertical velocity budget in cloudy updrafts

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FIGURE 9.—Photograph of seeded cloud 1, July 28, 1965, at seeding time (2217:30 GMT). Right-hand portion seeded.

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The parameterized vertical velocity equation

$$\frac{dw}{dt} = w\frac{dw}{dz} = \frac{d}{dz}\left(\frac{w^2}{2}\right) = \frac{gB}{1+\gamma} - \frac{3}{8}\left(\frac{3}{4}K_2 + C_D\right)\frac{w^2}{R}$$

γ	virtual mass coefficient
K_2	the entrainment coefficient
C_d	aerodynamic drag coefficient
R	cloud radius

$$\frac{1}{M_c}\frac{\partial M_c}{\partial z} = \frac{9}{32}\frac{K_2}{R}$$

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TABLE 1.—*Parameters of the EMB cumulus models*

Parameter	Meaning	EMB 65	EMB 68	Remarks
K_2	Entrainment	0.55	0.65	Lab. value 0.71
C_D	Aerodynamic drag coefficient	0.506	0	Solid-sphere value=1.125
γ	Virtual mass coefficient	0	0.5	Lab. value 0.5
LWC	Liquid water retained	½ condensate	Falloutscheme	Much improved in EMB 68

The parameterized vertical velocity equation

Vertical Velocity Equation	Constants	Source	Eq.
$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = aB_s - 1.45\frac{w_s^2}{R}$	$a = \frac{2}{3}, R = ?$	Simpson and Wiggert (1969)	(1)
$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = aB_s - (b\delta + \epsilon)w_s^2$	$a = 1/6, b = 1/2$	Gregory (2001)	(11)
$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = aB_s - b\epsilon w_s^2$	$a = 1, b = 2$	Bretherton et al. (2004)	(17)
$\frac{1}{2}(1-2\mu)\frac{\partial w_s^2}{\partial z} = aB_s - b\epsilon w_s^2$	$a = 1, b = 1/2, \mu = 0.15$	Siebesma et al. (2007)	(15)

in the literature different values for *a* and *b* are proposed

Large difference in proposed values for constants

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a'	b'
2/3	?
1/3	3
1	2
10/7	5/7

The equations in the Table can all be expressed as:
(Gregory: only if σ(z) = cst)

$$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = a'B_s - b'\epsilon w_s^2.$$

Derivation of the conditionally sampled vertical velocity equation

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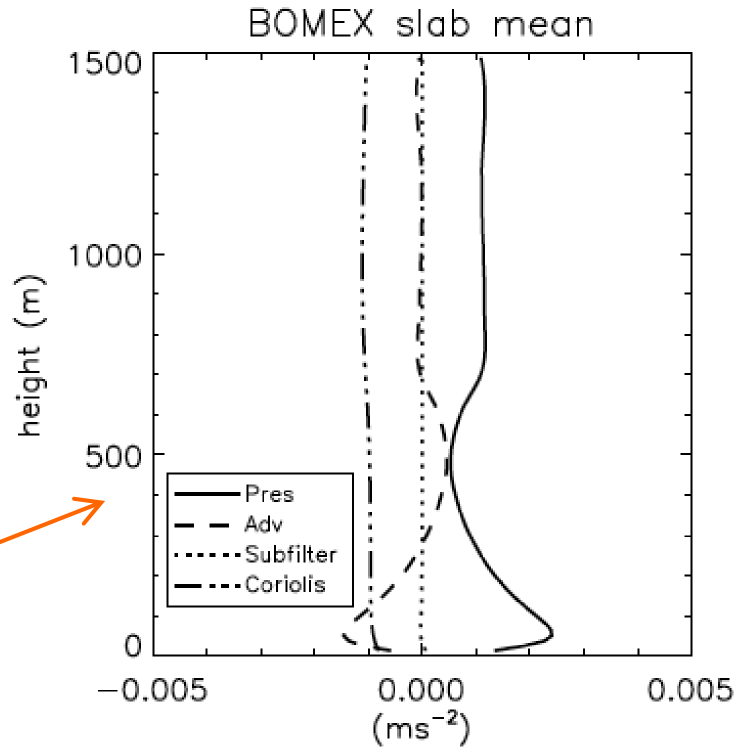
$$\underbrace{\frac{\partial w}{\partial t}}_{\text{tendency}} = -\underbrace{\frac{\partial u_j w}{\partial x_j}}_{\text{advection}} - \underbrace{\frac{\partial \overline{u_j''w''}}{\partial x_j}}_{\text{subgrid}} + \underbrace{\frac{g}{\theta_0}\theta_v}_{\text{buoyancy}} - \underbrace{\frac{1}{\rho_0}\frac{\partial P}{\partial z}}_{\text{pressure}} + \underbrace{\left(2\Omega\cos\varphi\right)u}_{\text{coriolis}}$$

Slab mean balance

$$\frac{\partial \overline{w}}{\partial t} = \frac{g}{\theta_0}\overline{\theta_v} - \frac{\partial \overline{ww}}{\partial z} - \frac{\partial \overline{\tau_{33}}}{\partial z} - \frac{1}{\rho_0}\frac{\partial \overline{P}}{\partial z} + 2\Omega \cos\varphi \overline{u}.$$

$$\frac{1}{\rho_0}\frac{\partial \overline{p}_{hyd}}{\partial z} \equiv \frac{g}{\theta_0}\overline{\theta_v}$$

$$p \equiv \frac{P}{\rho_0} - \frac{\overline{p}_{hyd}}{\rho_0} + \frac{2}{3}e$$



Sampling operator

$$[\psi]_s(z) = \frac{\int_A I_s(x,y,z)\psi(x,y,z)dA}{\int_A I_s(x,y,z)dA}$$

$$\left[\frac{\partial w}{\partial t}\right]_s = -\left[\frac{\partial ww}{\partial z}\right]_s - \left[\frac{\partial u_j w}{\partial x_j}\right]_s - \left[\frac{\partial \tau_{33}}{\partial z}\right]_s - \left[\frac{\partial \tau_{3j}}{\partial x_j}\right]_s + [S_w]_s$$

Leibniz rule

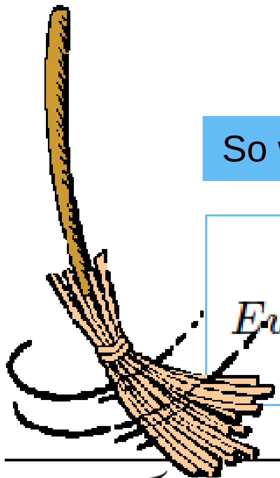
$$\left[\frac{\partial w}{\partial t}\right]_s = \frac{\partial w_s}{\partial t} + \frac{w_s}{\sigma}\frac{\partial \sigma}{\partial t} + \left\{\frac{\partial w}{\partial t}\right\}_L$$

Leibniz for tendency

$$\frac{\partial \sigma w_s}{\partial t} = -\frac{\partial \sigma w_s w_s}{\partial z} - \frac{\partial \sigma \overline{w''w''}^s}{\partial z} + E w_e - D w_s + \sigma \left[S_w\right]_s$$

Siebesma and Cuijpers (1995)
but ϕ replaced by *w*

So what's in the lateral exchange term?



$$E w_e - D w_s = -\sigma\left[\frac{\partial u_j w}{\partial x_j}\right]_s - \sigma\left[\frac{\partial \tau_{3j}}{\partial x_j}\right]_s - \sigma\left\{\frac{\partial w}{\partial t}\right\}_L - \sigma\left\{\frac{\partial w w}{\partial z}\right\}_L - \sigma\left\{\frac{\partial \tau_{33}}{\partial z}\right\}_L$$

Massflux equation

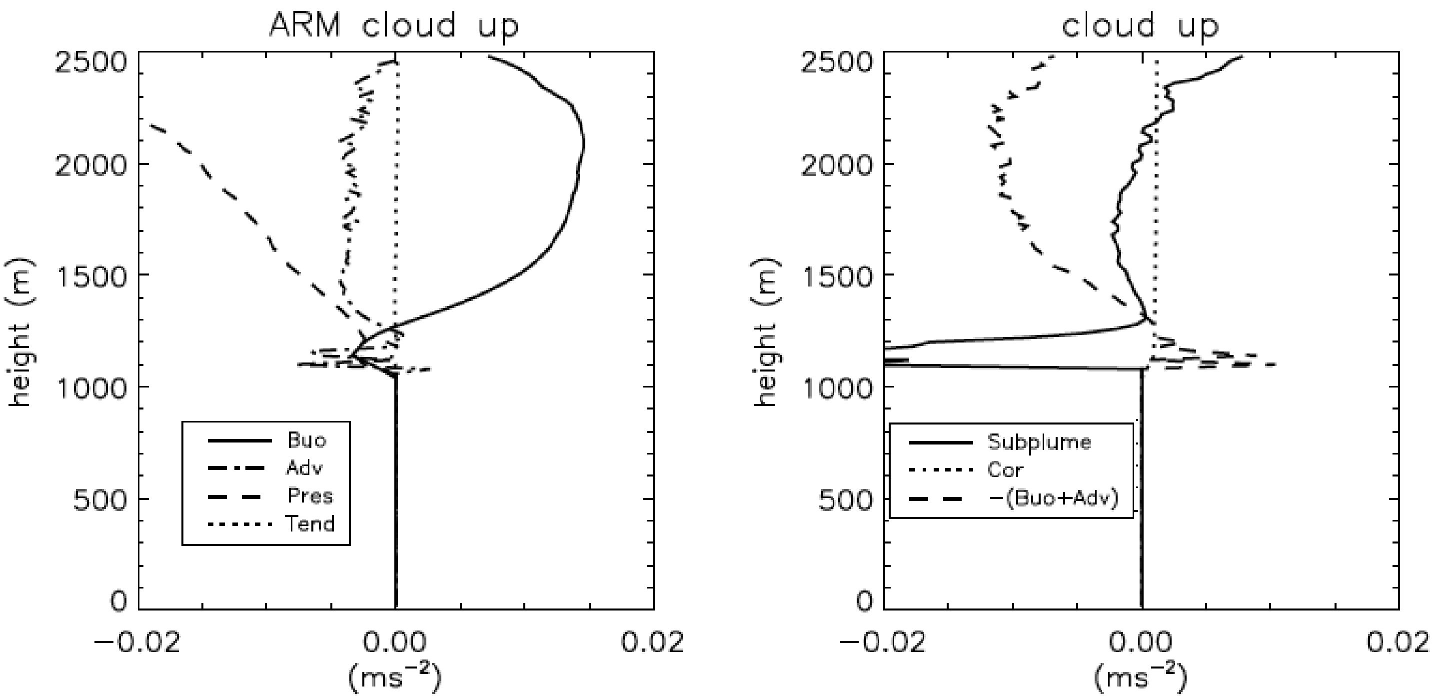
$$\frac{\partial M_c}{\partial z} = -\frac{\partial \sigma}{\partial t} + E - D$$

Substitution:

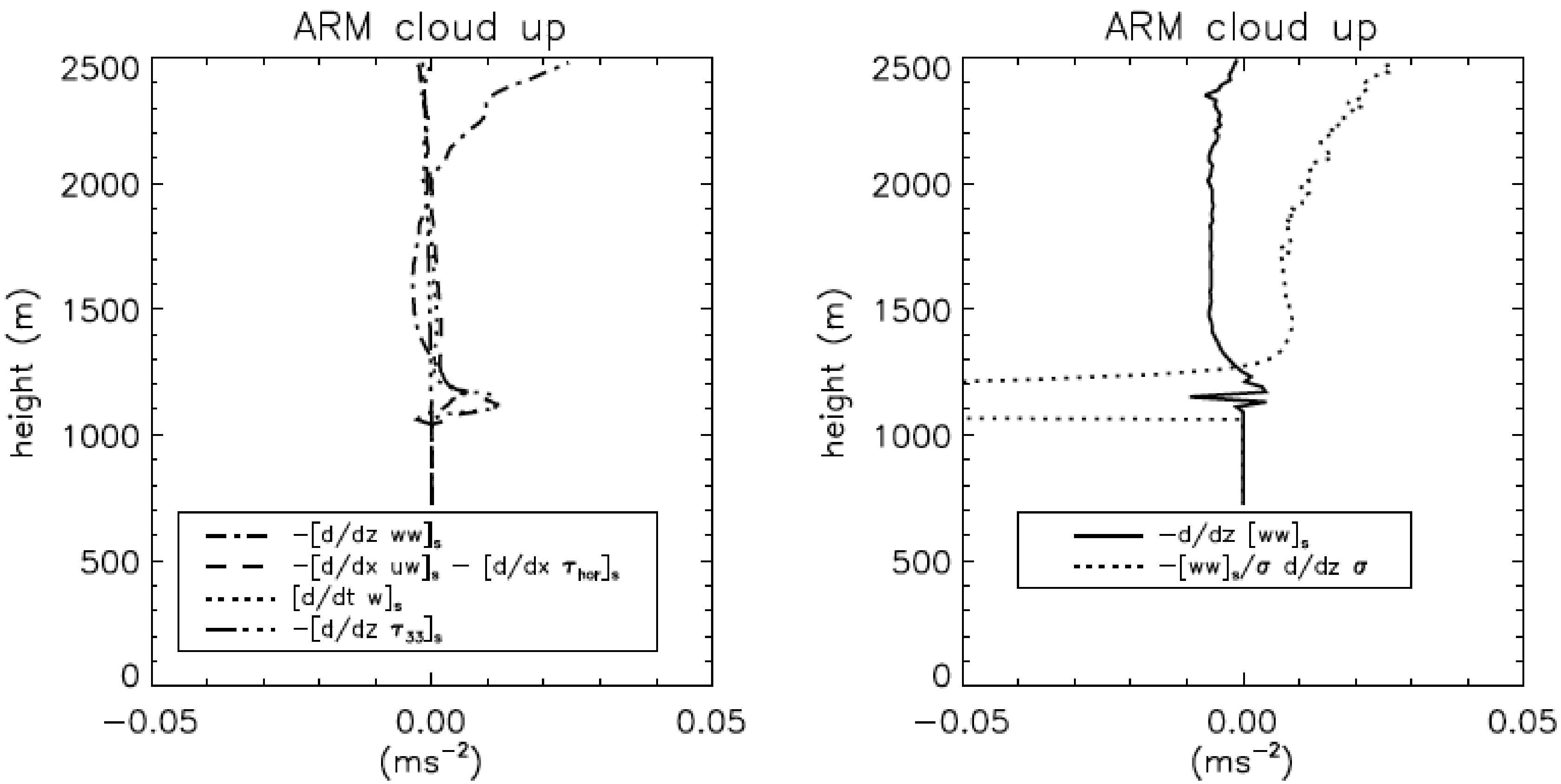
$$\underbrace{\frac{\partial w_s}{\partial t}}_{\text{Tend}} = \underbrace{B_s}_{\text{Buo}} - \underbrace{\frac{1}{2}\frac{\partial w_s^2}{\partial z}}_{\text{Adv}} - \underbrace{\frac{\epsilon w_s^2}{1-\sigma}}_{\text{Ent}} - \underbrace{\frac{1}{\sigma}\frac{\partial \sigma \overline{w''w''}^s}{\partial z}}_{\text{Subplume}} - \underbrace{\left[\frac{\partial p}{\partial z}\right]_s}_{\text{Pres}} + \underbrace{2\Omega \cos \varphi u_s}_{\text{Cor}}$$

w budgets for ARM: term –(Buo+Adv) ~ -bεw_s^2 (but actually represents effect of pressure damping)

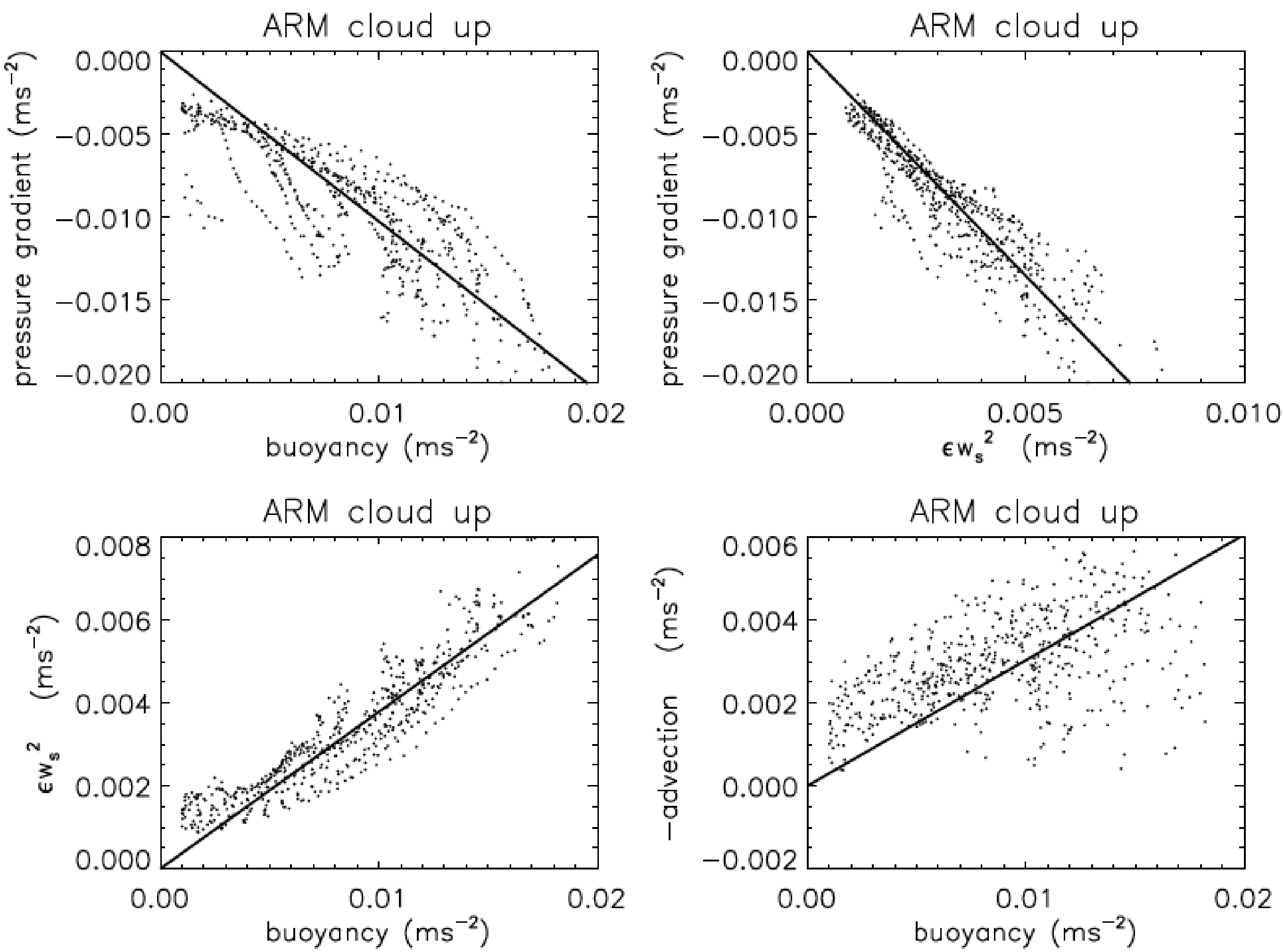
$$\underbrace{\frac{\partial w_s}{\partial t}}_{\text{Tend}} = \underbrace{B_s}_{\text{Buo}} - \underbrace{\frac{1}{2}\frac{\partial w_s^2}{\partial z}}_{\text{Adv}} - \underbrace{\frac{\epsilon w_s^2}{1-\sigma}}_{\text{Ent}} - \underbrace{\frac{1}{\sigma}\frac{\partial \sigma \overline{w''w''}^s}{\partial z}}_{\text{Subplume}} - \underbrace{\left[\frac{\partial p}{\partial z}\right]_s}_{\text{Pres}} + \underbrace{2\Omega \cos\varphi u_s}_{\text{Cor}}$$



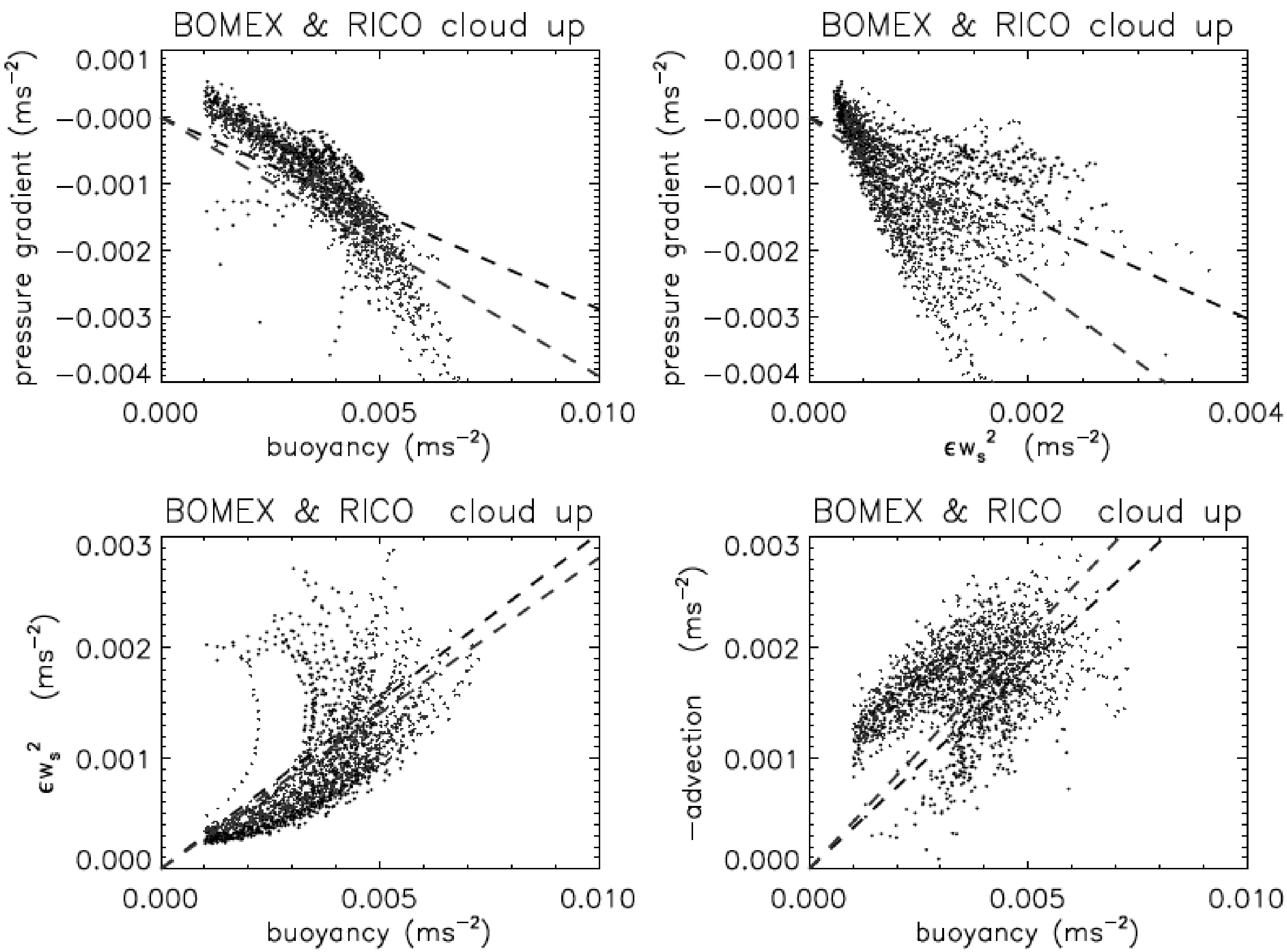
w budgets


ARM cloud up
2500
2000
1500
1000
500
0
height (m)
−0.05
0.00
0.05
(ms^-2)
−[d/dz ww]s
−[d/dx uw]s − [d/dx τhor]s
[d/dt w]s
−[d/dz τ33]s
ARM cloud up
2500
2000
1500
1000
500
0
height (m)
−0.05
0.00
0.05
(ms^-2)
−d/dz [ww]s
−[ww]s/σ d/dz σ

scatter plots: ARM



scatter plots: BOMEX and RICO



Budgets do not exhibit universal scaling behavior

	Cloud core			Cloud updraft		
	BOMEX	ARM	RICO	BOMEX	ARM	RICO
$-\left[\frac{\partial p}{\partial z}\right]_s = \alpha_p B_s$	-0.58	-1.06	-0.53	-0.39	-1.02	-0.29
$-\left[\frac{\partial p}{\partial z}\right]_s = p_\epsilon \epsilon w_s^2$	-3.35	-5.17	-2.19	-1.23	-2.71	-0.76
$\epsilon w_s^2 = \alpha_\epsilon B_s$	0.15	0.19	0.16	0.28	0.38	0.30
$\frac{\partial w_s^2}{\partial z} = \eta B_s$	0.35	0.23	0.32	0.42	0.30	0.37

Table 2: Fit coefficients α_p, p_ϵ, α_ϵ, and η of a linear regression trough origin for the scatter plots

Discussion

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$$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = a'B_s - b'\epsilon w_s^2$$

→

$$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = (a' - b'\alpha_\epsilon)B_s \equiv \eta B_s$$

De Rooij and Siebesma propose $\alpha_\varepsilon B_s = \varepsilon w_s^{\ 2}$

(diluted) CAPE: $\eta/2 \sim 0.15\text{-}0.2$

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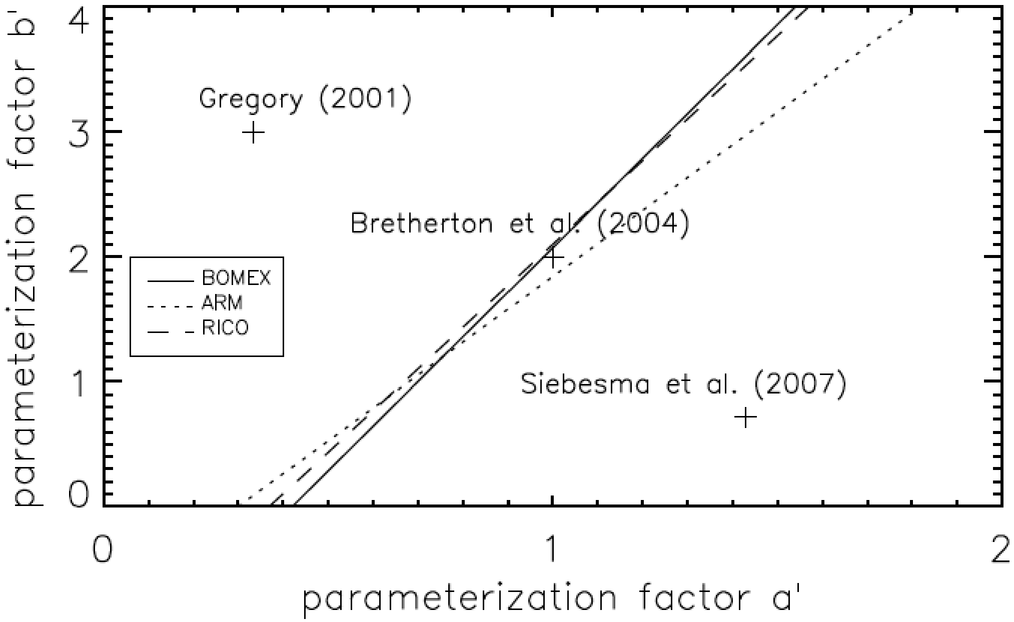
$$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = \frac{\eta}{2} B_s << B_s$$

Consequence: b' and a' are dependent

$$\frac{1}{2}\frac{\partial w_s^2}{\partial z} = (a' - b'\alpha_\epsilon)B_s \equiv \eta B_s$$

$\longrightarrow$

$$b' = \frac{a' - \eta}{\alpha_\epsilon}$$



Conclusions

1. No universal scaling behavior is found
2. Pressure term dominant destruction term
3. Only small fraction of (diluted) CAPE used for producing vertical motions
4. a' and b' are dependent factors
5. more budgets shown in De Roode et al. (2011)