# Radiative-Convective Instability

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# Program

- **Basic radiative-convective equilibrium**
- **Macro-instability of the RC state**
- **Some consequences**

# Radiative Equilibrium

- Equilibrium state of atmosphere and surface in the absence of non-radiative enthalpy fluxes
- Radiative heating drives actual state toward state of radiative equilibrium

# Terrestrial Radiation:

Effective emission temperature, $T_e$:

Solar constant

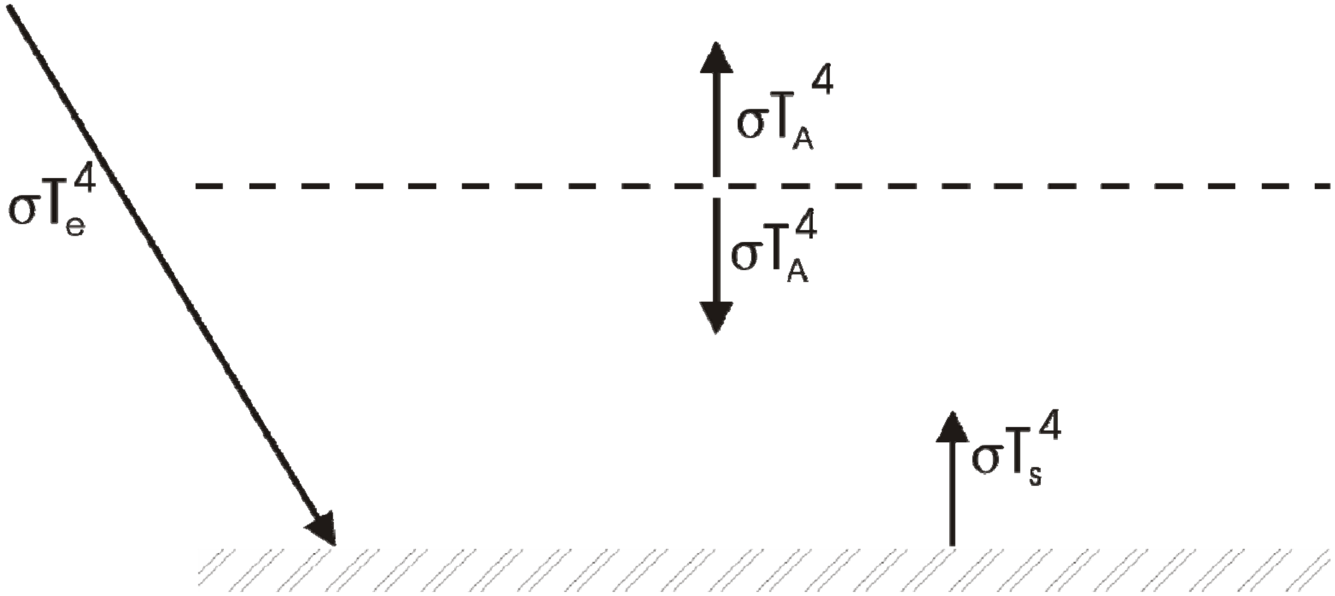
$$\sigma T_e^4 \equiv \frac{S_0}{4}\left(1-a_p\right)$$

Planetary albedo

Earth: $T_e = 255K = -18^\circ C$

Observed average surface temperature $= 288K = 15^\circ C$

# One-Layer Model



- Transparent to solar radiation
- Opaque to infrared radiation
- Blackbody emission from surface and each layer

# Radiative Equilibrium:

Top of Atmosphere:

$$\sigma T_A^{\,4} = \frac{S_0}{4}\left(1 - a_p\right) = \sigma T_e^{\,4}$$

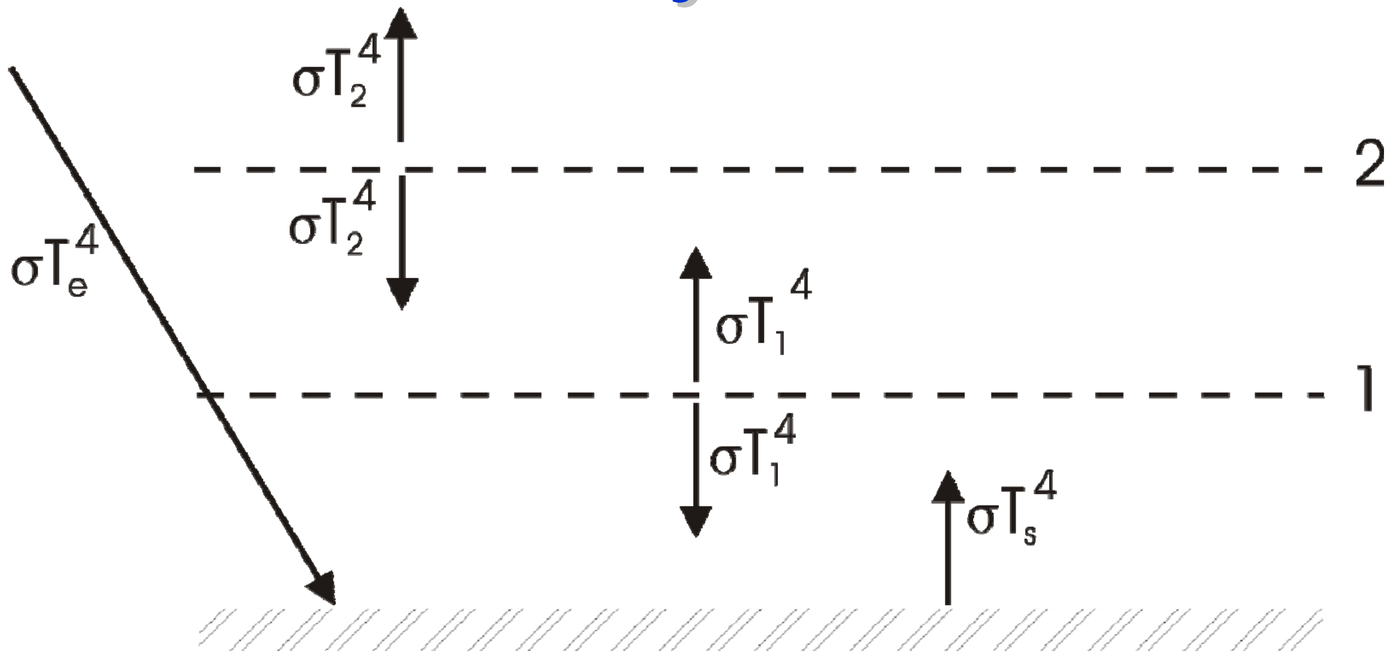
$$\rightarrow \quad \boxed{T_A = T_e}$$

Surface:

$$\sigma T_s^{\,4} = \sigma T_A^{\,4} + \frac{S_0}{4}\left(1 - a_p\right) = 2\sigma T_e^{\,4}$$

$$\rightarrow \boxed{T_s = 2^{1/4} T_e} = 303\,K$$

# Two-Layer Model



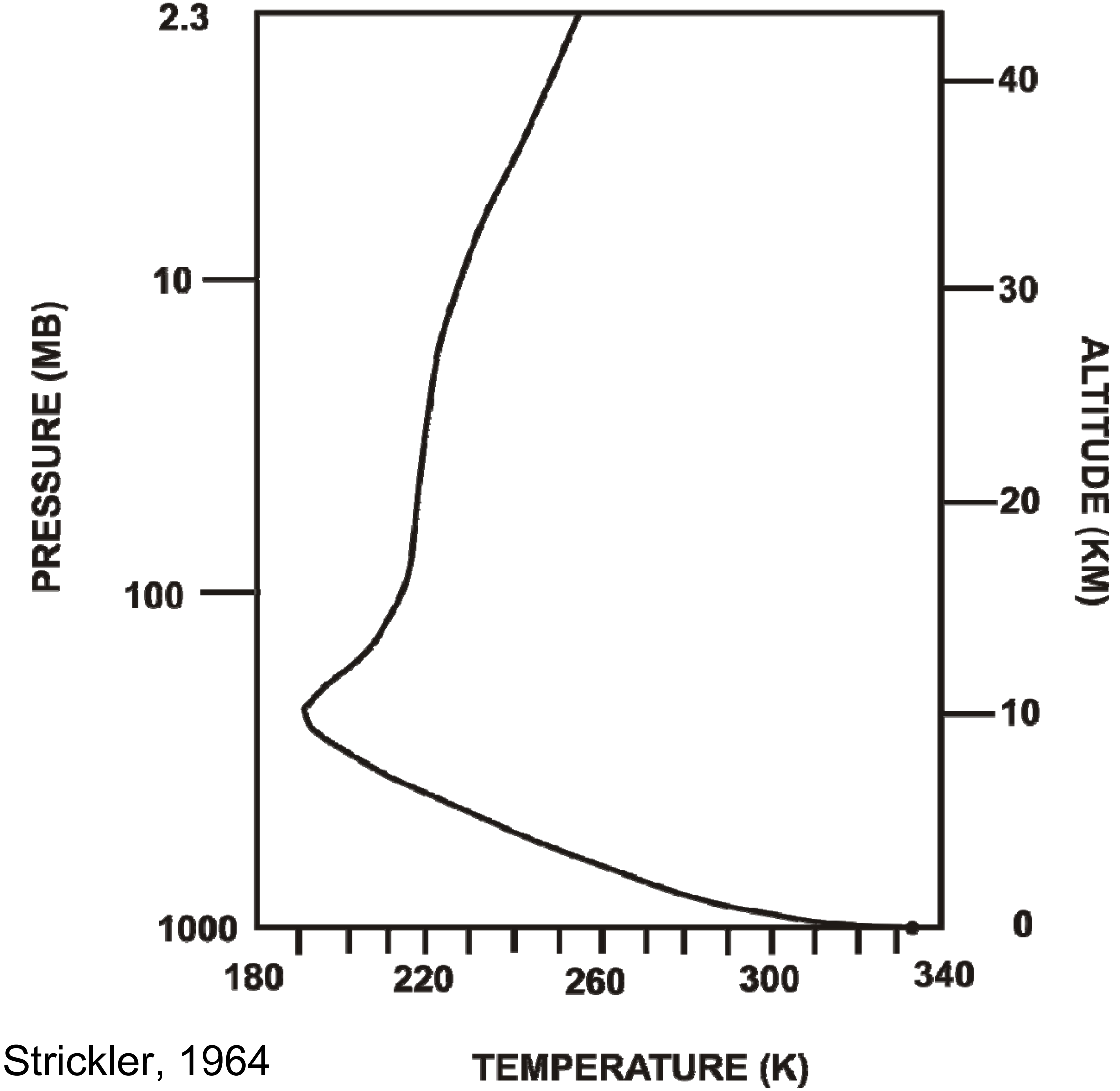
$$TOA: \quad \sigma T_2^{\,4} = \sigma T_e^{\,4} \rightarrow T_2 = T_e$$

$$Middle\ Layer: \quad 2\sigma T_1^{\,4} = \sigma T_2^{\,4} + \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_s^{\,4}$$

$$Surface: \quad \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_1^{\,4}$$

$$\rightarrow \quad T_s = 3^{1/4} T_e \qquad\qquad T_1 = 2^{1/4} T_e$$

# Full calculation of radiative equilibrium:



After Manabe and Strickler, 1964

# Problems with radiative equilibrium solution:

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)

→ **Troposphere is unstable to moist convection**

# When is an atmosphere unstable to convection?

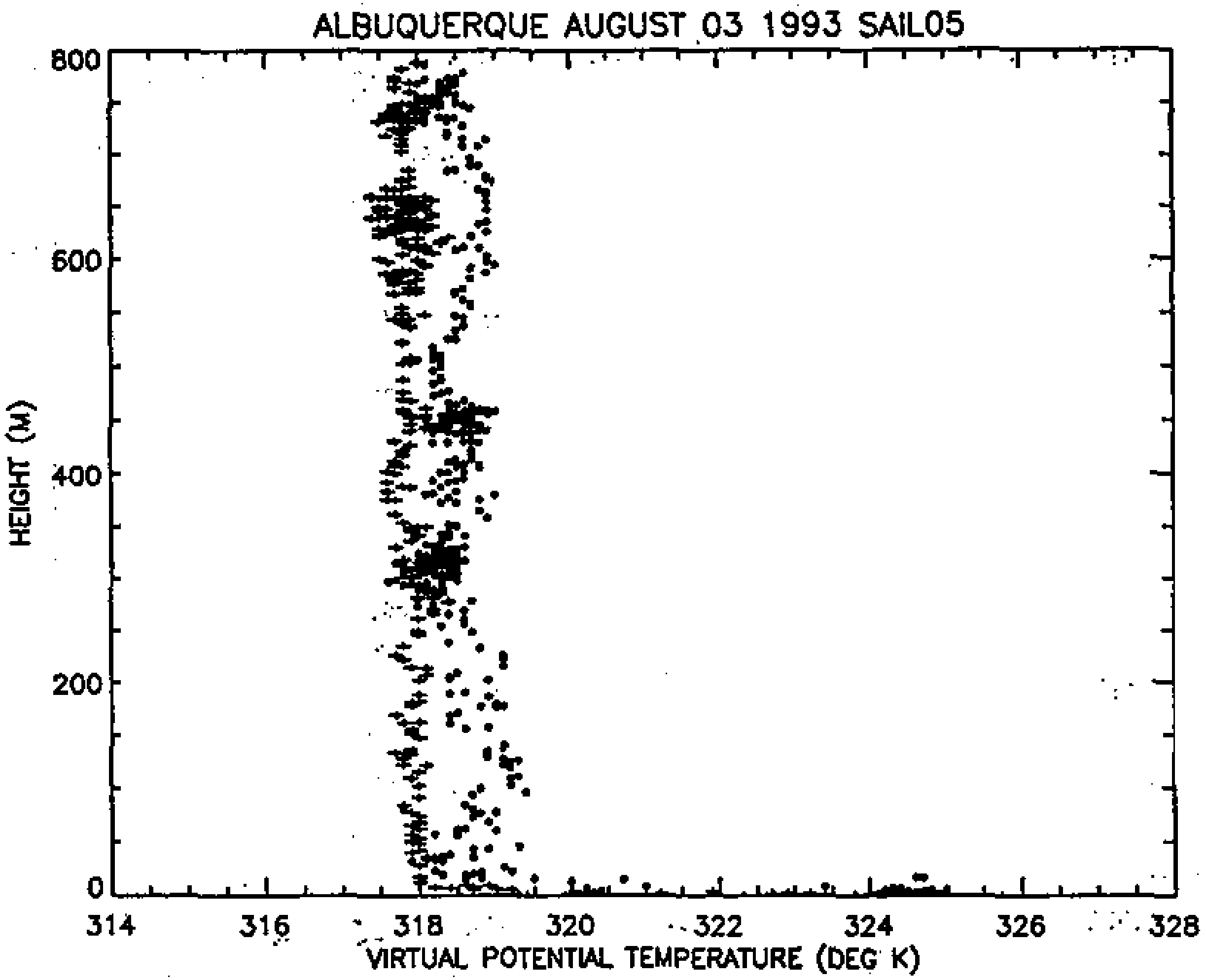
- Dry atmosphere: When entropy decreases upward. This is true of pure radiative equilibrium

# Radiative-Convective Equilibrium:

## Statistical equilibrium state in which radiative and convective fluxes sum to zero

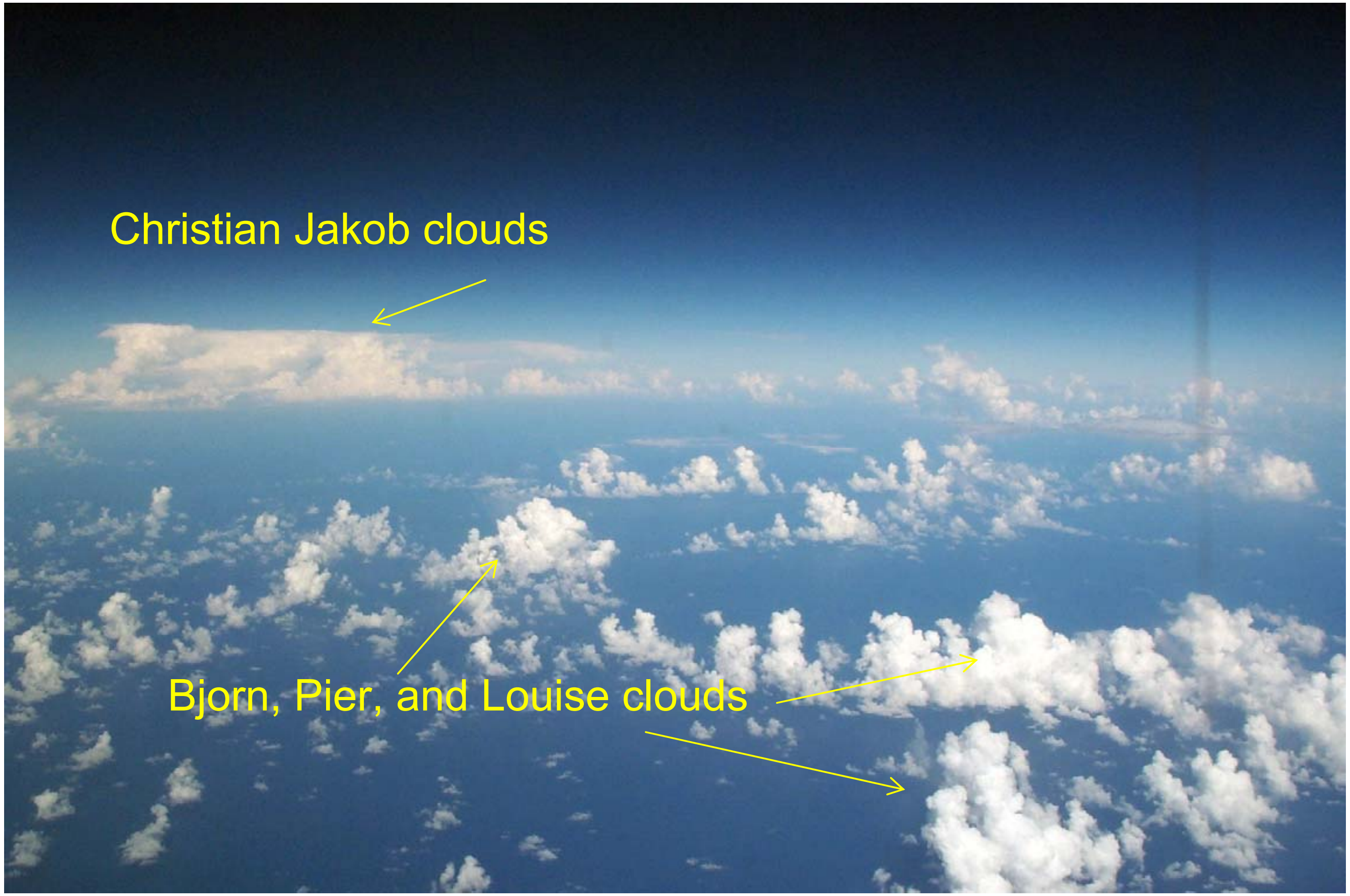
- Radiative relaxation time scales ~ 40 days
- Convective adjustment time scales:  minutes (dry) to hours (moist)
- In competition between radiation and convection, convections “wins” and the observed state is much closer to convective neutrality than to radiative equilibrium

# Dry convective boundary layer over daytime desert (Renno and Williams, 1995)



But above a thin boundary layer, most atmospheric convection involves phase change of water:

# Moist Convection



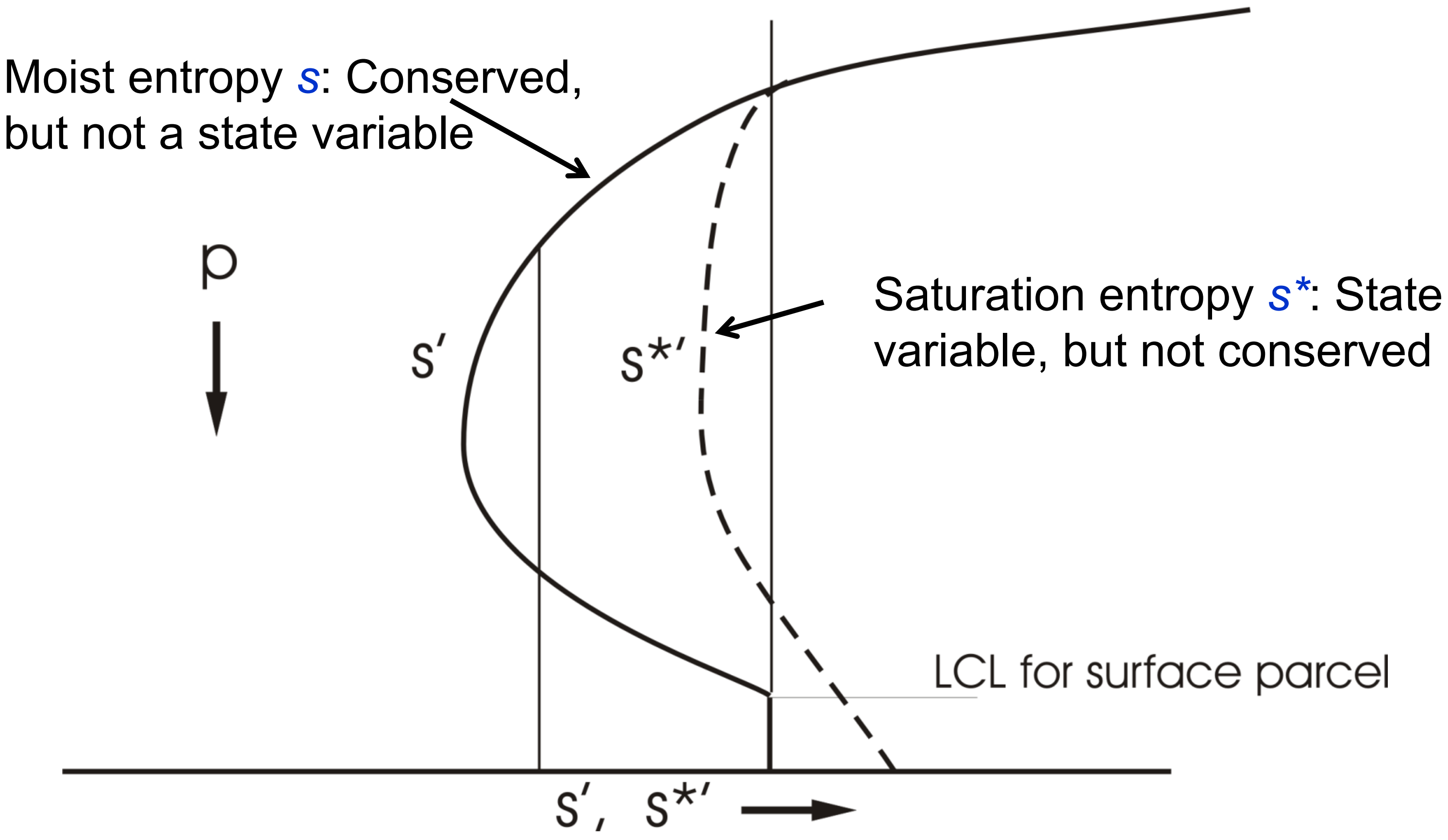
# Moist Convection

- Significant heating/cooling owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping

# Radiative-Moist Convective Equilibrium

- Vertical profile nearly neutral to moist convection

- Strongly-two way interaction: Radiation drives profile toward instability, convection lofts water that strongly affects radiative transfer



Moist entropy *s*: Conserved, but not a state variable

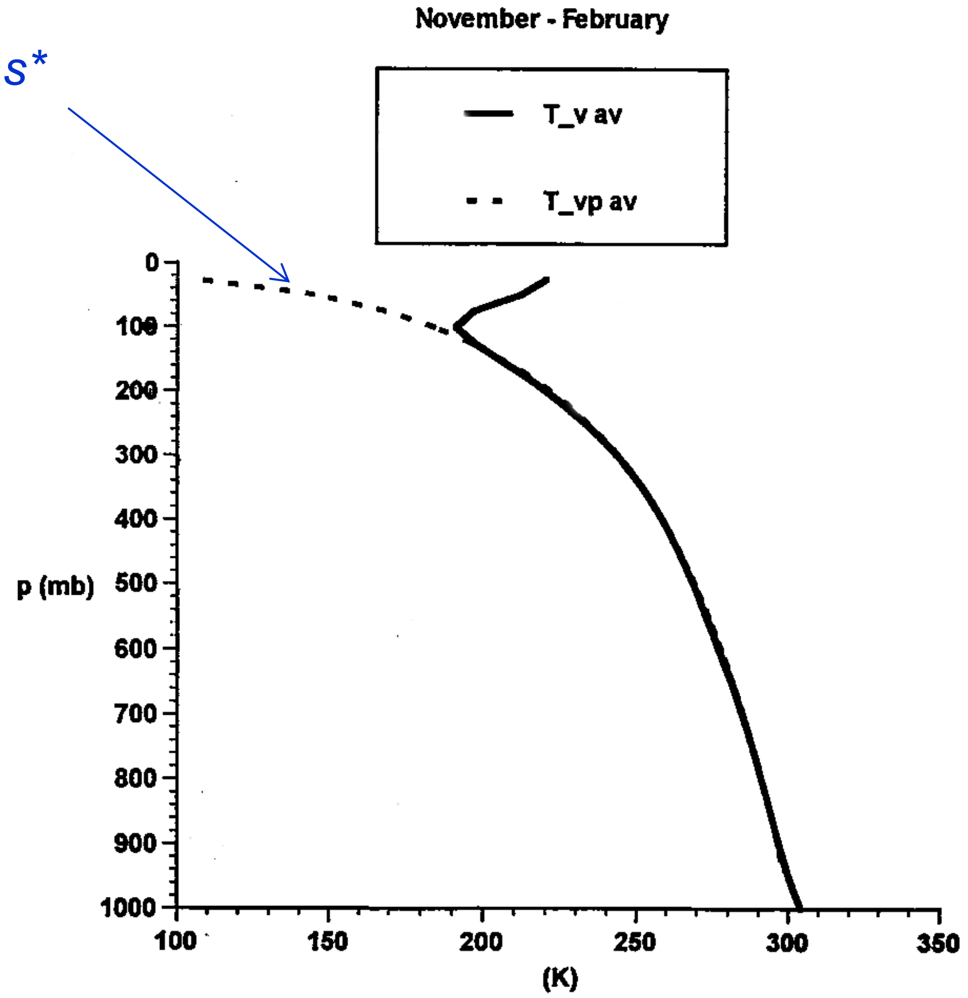
Saturation entropy *s**: State variable, but not conserved

$$s \equiv \left(c_p\left(1-q_t\right)+c_w q_t\right)\log(T)+\frac{L_v q_v}{T}-R_d\left(1-q_t\right)\log(p_d)-q_v R_v \log\left(e/e^*\right)$$

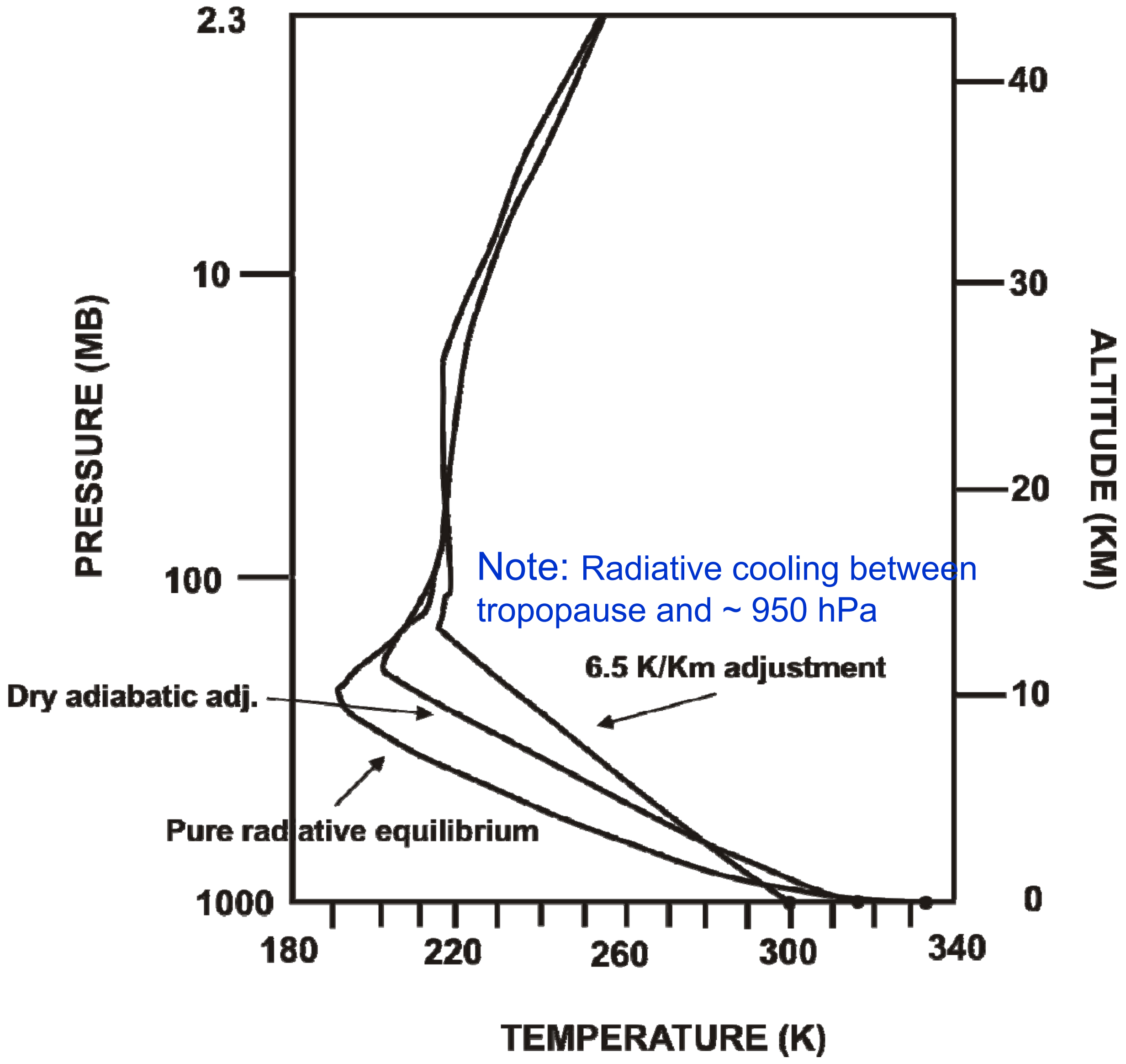
$$s^* \equiv \left(c_p\left(1-q_t\right)+c_w q_t\right)\log(T)+\frac{L_v q_v^*}{T}-R_d\left(1-q_t\right)\log(p_d)$$

# Tropical Soundings

constant *s**



# Manabe and Strickler 1964 calculation:



Note: Radiative cooling between tropopause and ~ 950 hPa

# Real Radiative-Convective Equilibrium is Strongly Two-Way Interactive

Vertical profile of humidity (and therefore climate) ***strongly*** depends on

- Entrainment-detrainment physics
- Cloud microphysics, including re-evaporation of precipitation

# Convection/Railway Terms:

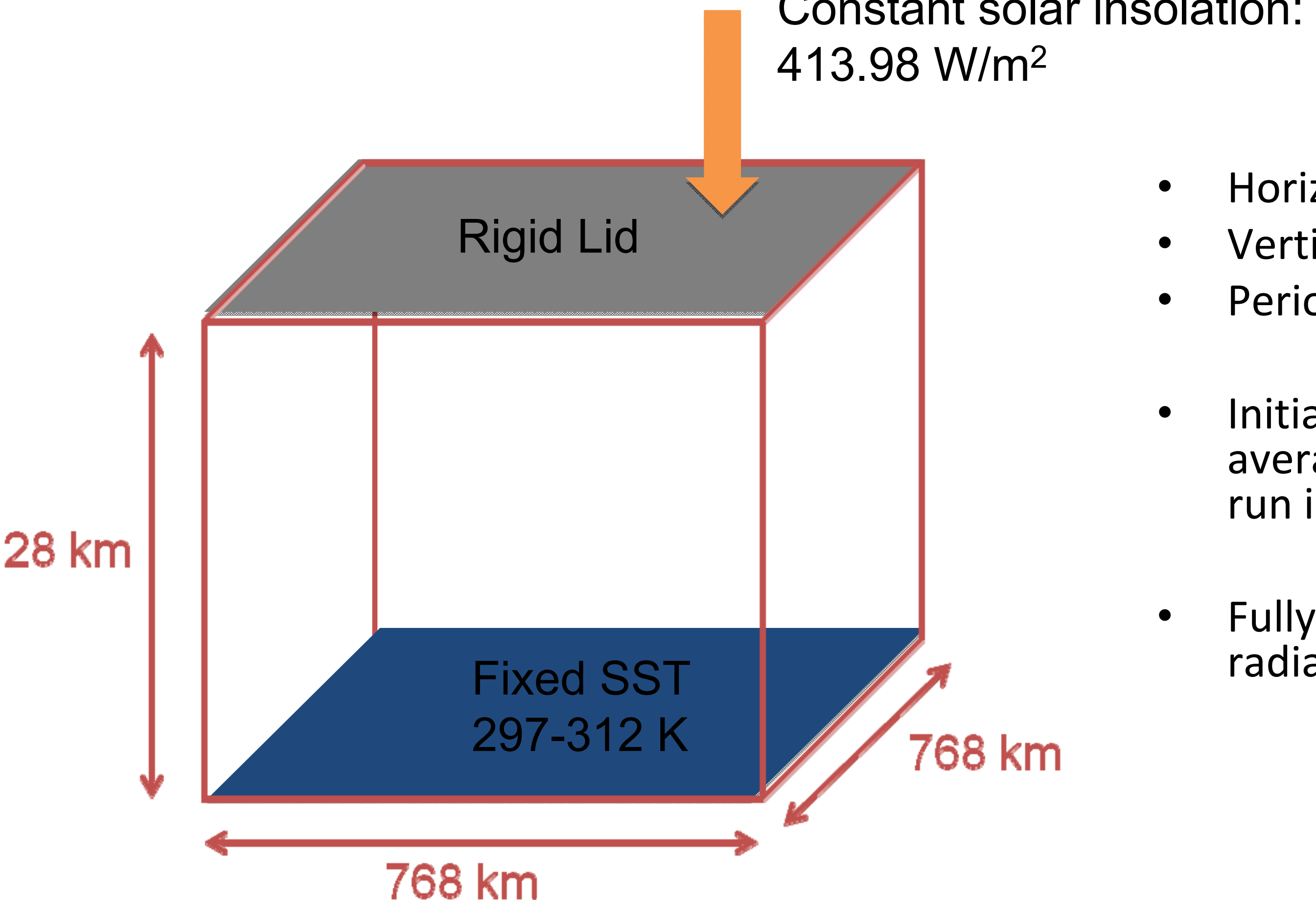
- ***Entrainment:*** Process by which quiescent environmental air becomes incorporated in the turbulent envelope of a cloud
- ***Detrainment:*** Process by which turbulent air considered part of cloud is ejected into and becomes part of the quiescent environment
- ***Derailment:*** What happens when a formerly productive researcher undertakes the study of entrainment and detrainment

# Explicit Simulation of Radiative-Convective Equilibrium

(Work of Allison Wing, and with much help from Marat Khairoutdinov)

# Approach: Idealized modeling of convective organization in radiative-convective equilibrium using a cloud resolving model

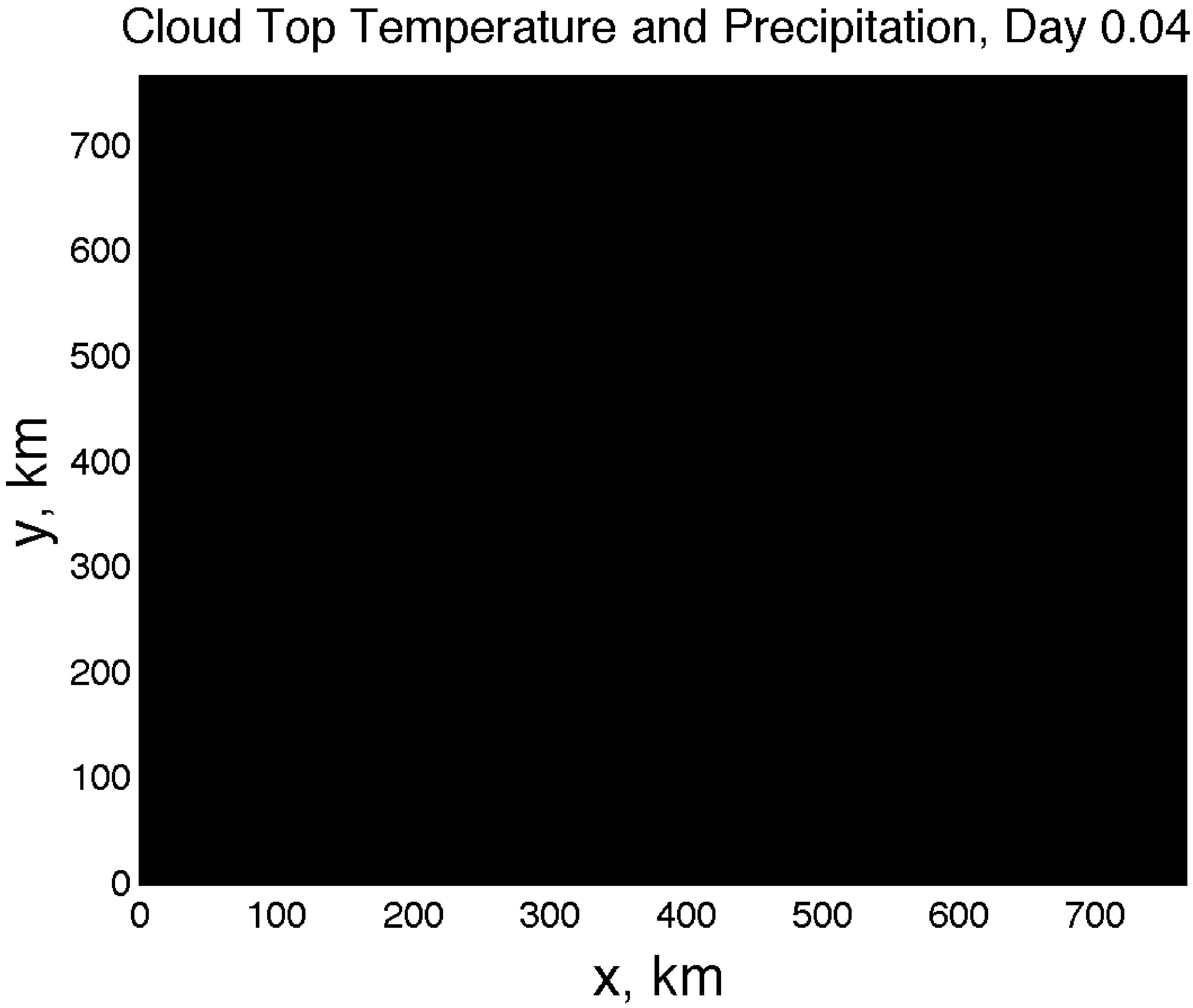
## System for Atmospheric Modeling (SAM) of Khairoutdinov and Randall (2003)



- Horizontal Resolution: 3km
- Vertical Resolution: 64 levels
- Periodic lateral boundaries

- Initial sounding from domain average of smaller domain run in RCE

- Fully interactive RRTM radiation and surface fluxes.


Cloud Top Temperature and Precipitation, Day 0.04
y, km
700
600
500
400
300
200
100
0
0
100
200
300
400
500
600
700
x, km

# Characteristics, including some surprises:

- As predicted by Bjerknes (1938), fractional area is small
- Net upward convective mass flux, *M*, constrained by

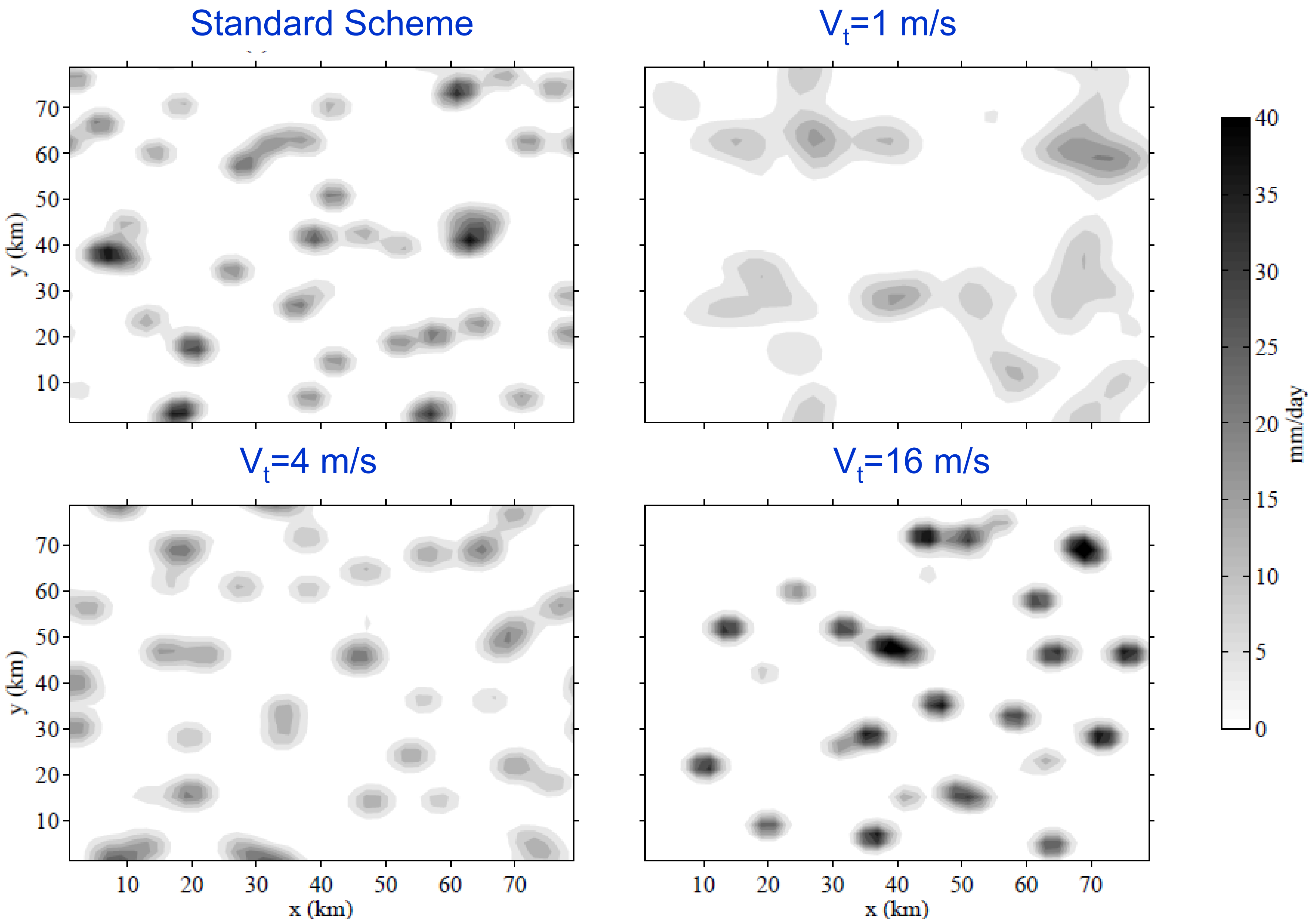
$$M\frac{c_p T}{\theta}\frac{\partial\theta}{\partial z} = -\dot{Q}_{rad}$$

fractional area

- But $M \sim \sigma w$... what determines $w$?
- Answer: Fall speed of precipitation!

# Daily rain accumulation

Courtesy Martin Singh



Standard Scheme

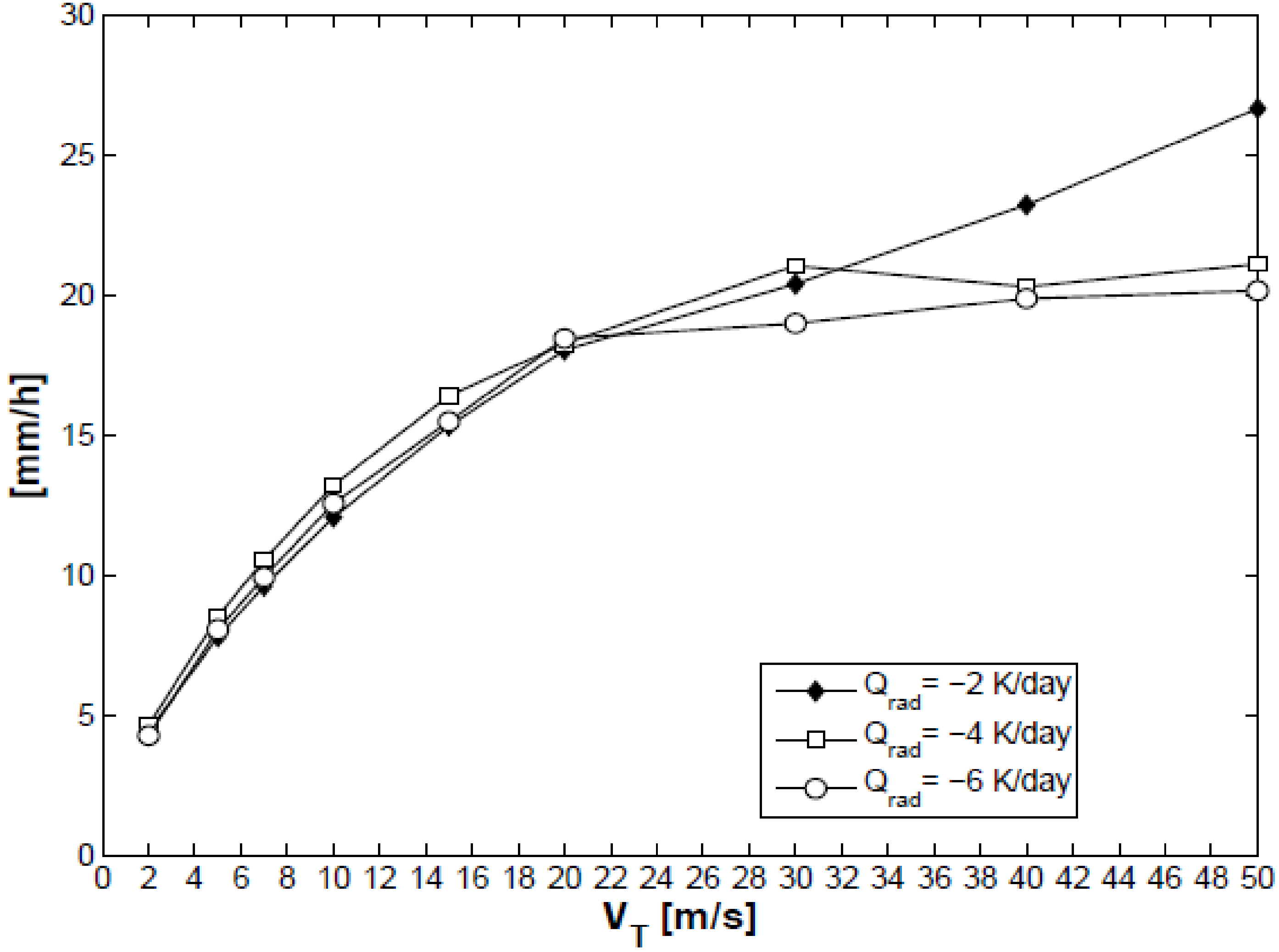
$V_t$=1 m/s

$V_t$=4 m/s

$V_t$=16 m/s

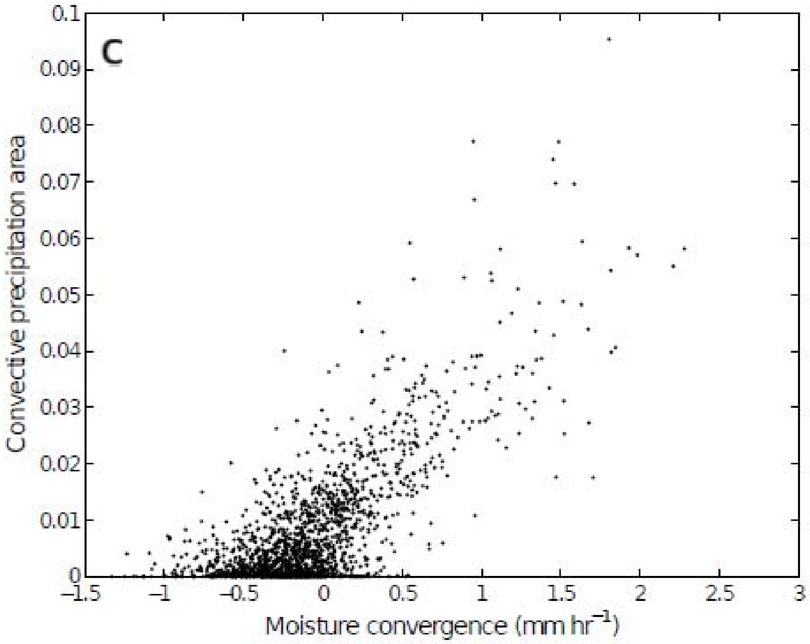
# Rainfall Intensity vs. Terminal Fall Speed

(Note: MEAN rainfall invariant, as radiative cooling is specified)

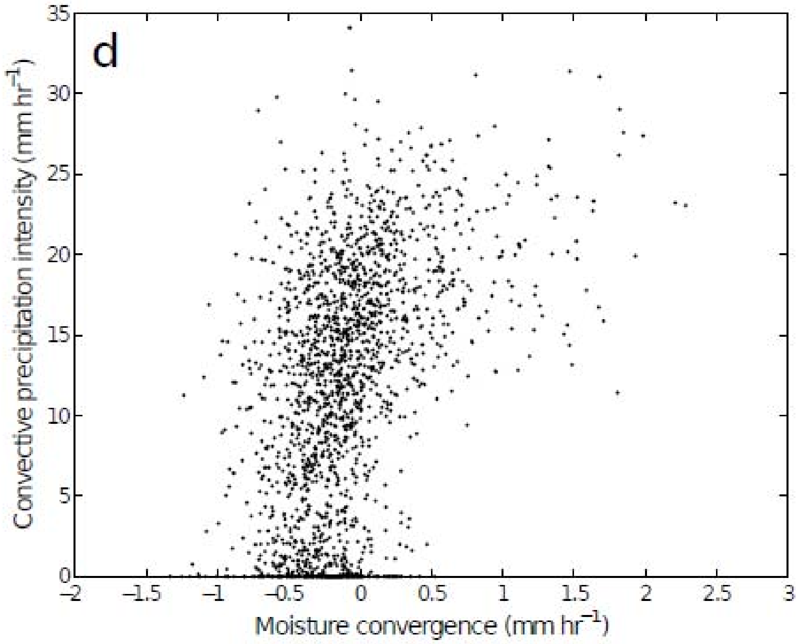


Parodi and Emanuel, 2009

# Comparison of Large-scale Moisture Convergence to Radar-Derived Convective Quantities (Davies et al., submitted)



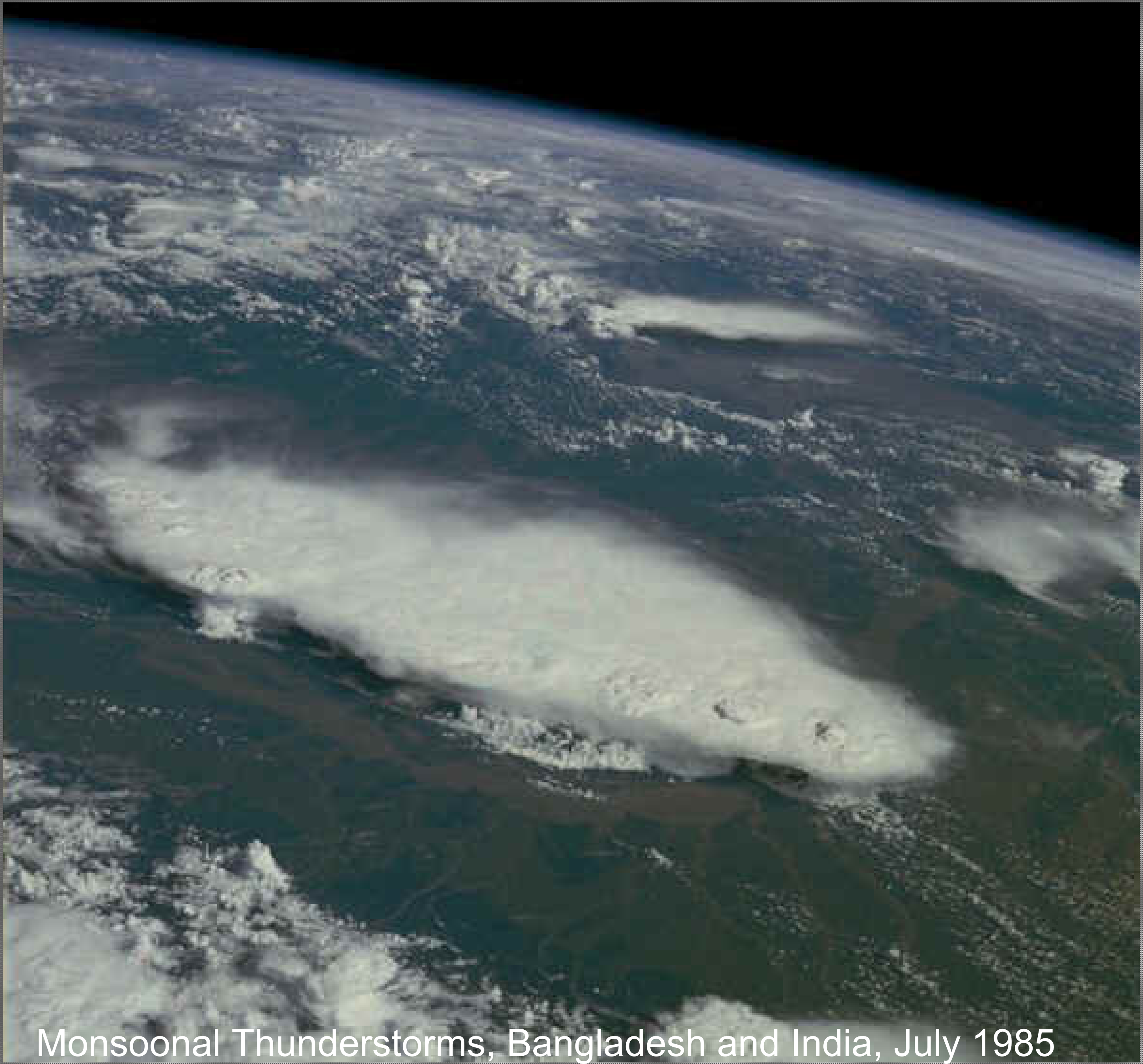
Convective precipitation



Convective precipitation intensity

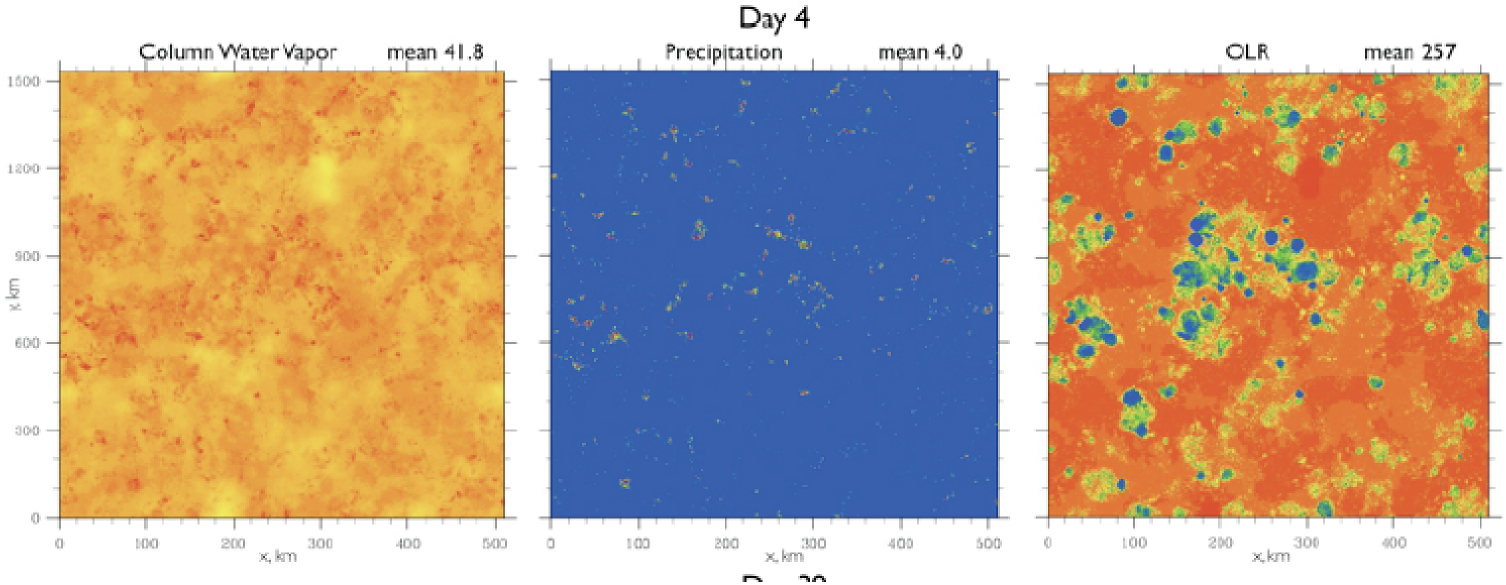
Courtesy Christian Jakob

# Radiative-Convective Instability: The Self-Aggregation of Convection

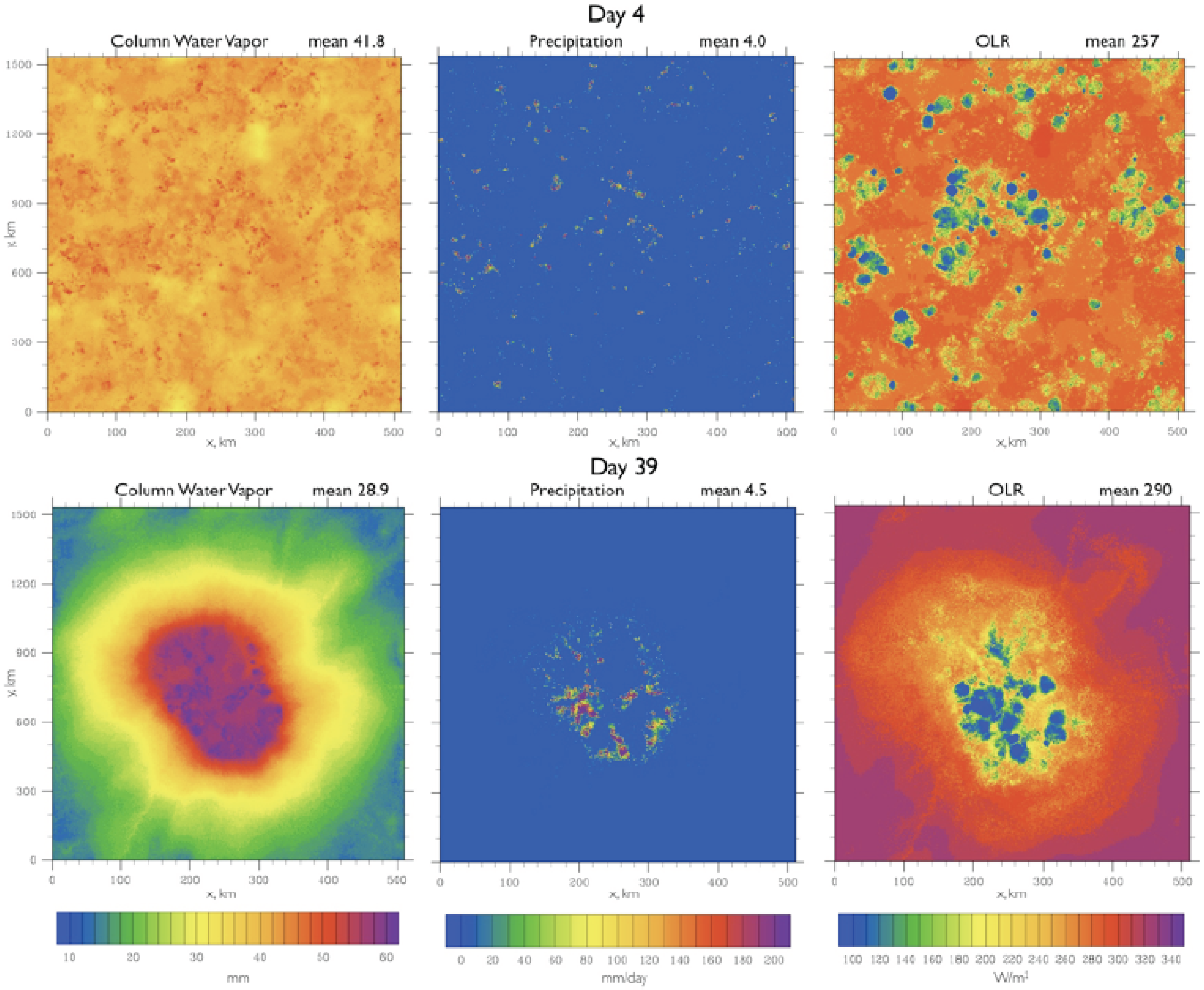


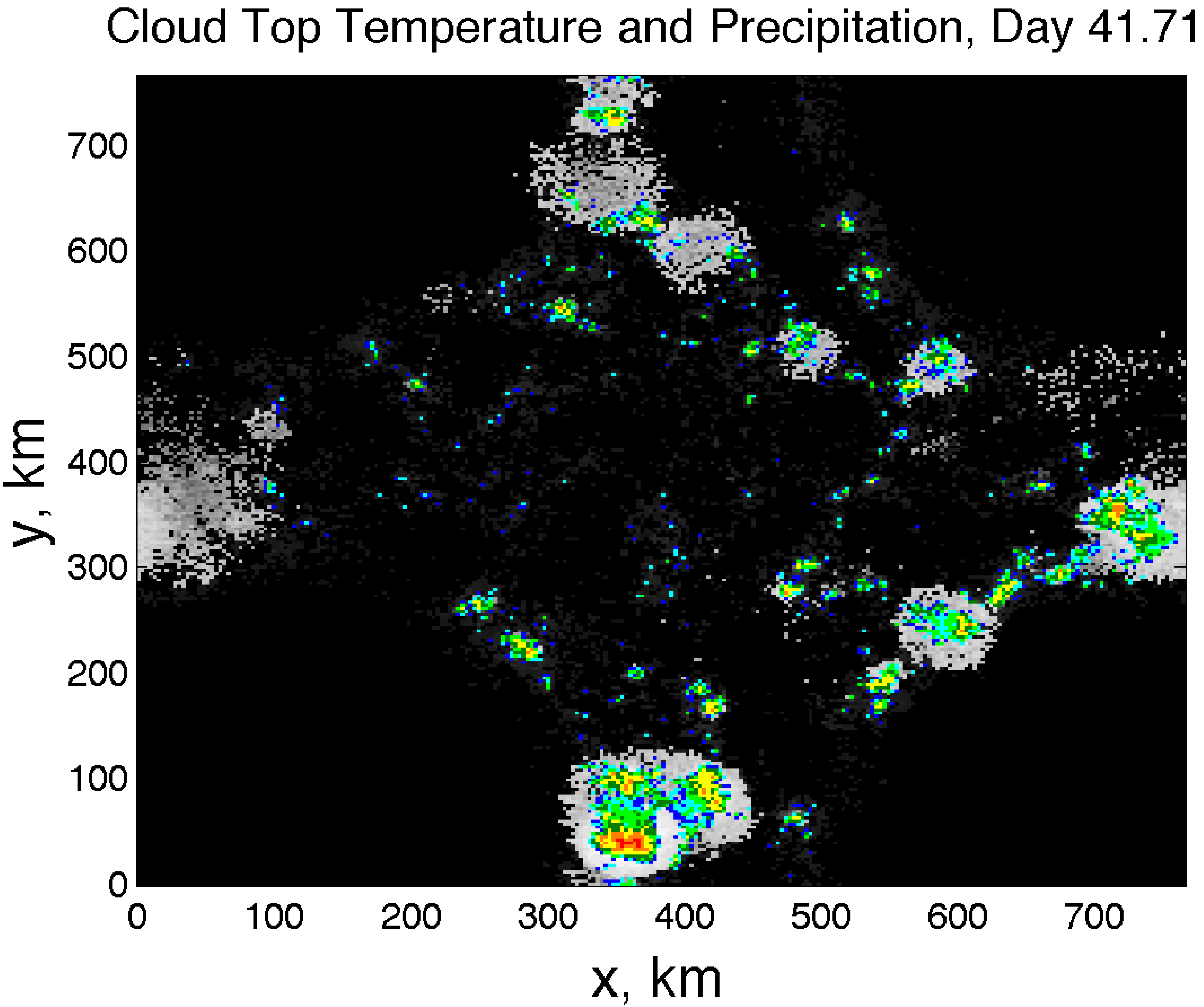
Monsoonal Thunderstorms, Bangladesh and India, July 1985

# Spontaneous Aggregation of Convection



# Spontaneous Aggregation of Convection




Cloud Top Temperature and Precipitation, Day 41.71
y, km
700
600
500
400
300
200
100
0
0
100
200
300
400
500
600
700
x, km

# Analysis of Feedback Terms

**Framework:** Budget for spatial variance of column integrated **frozen moist static energy**

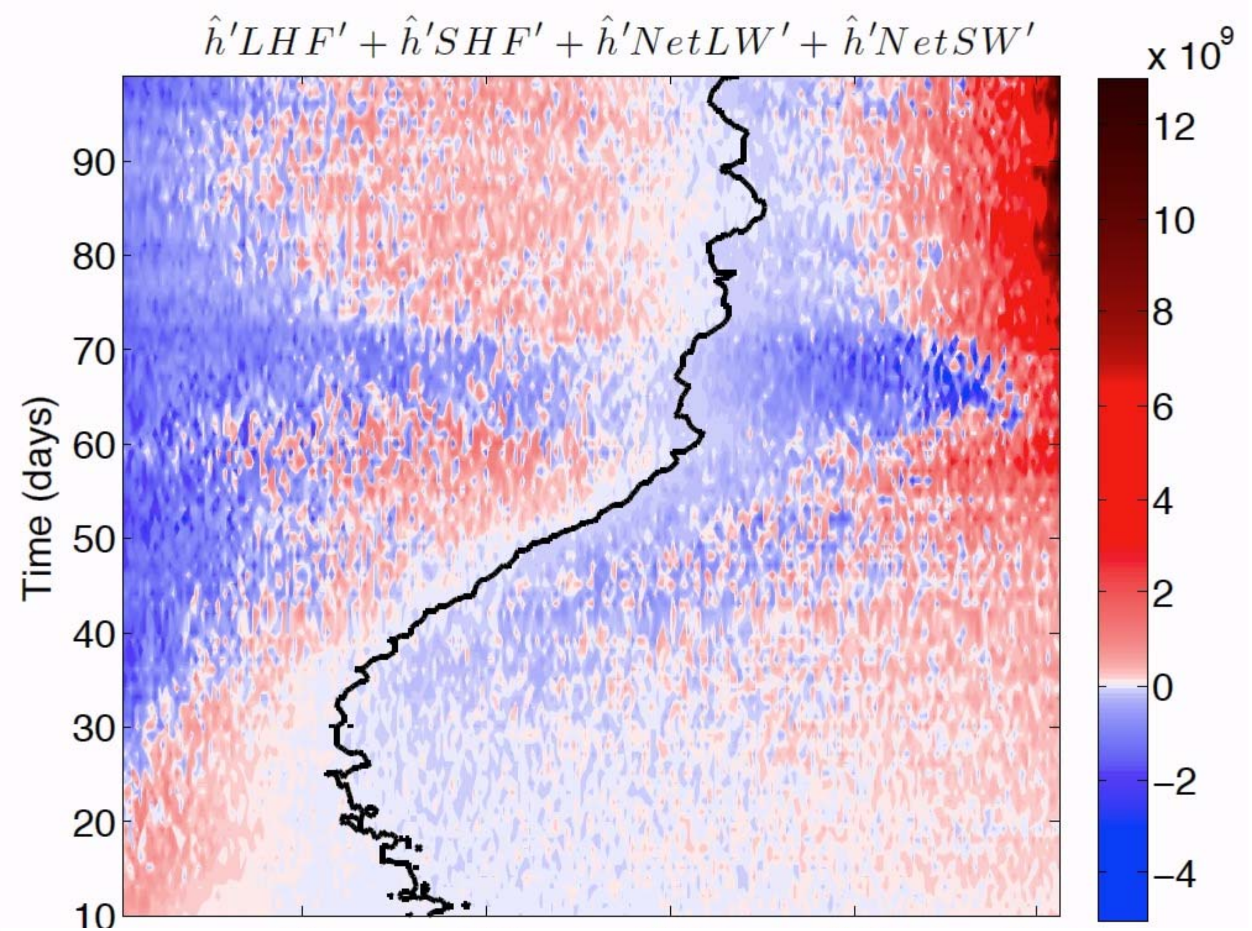
- Consider anomalies from the horizontal mean (primes)

$$\frac{1}{2}\frac{\partial \hat{h}'^2}{\partial t} = \hat{h}'\mathrm{LHF}' + \hat{h}'\mathrm{SHF}' + \hat{h}'\mathrm{NetSW}' + \hat{h}'\mathrm{NetLW}' - \hat{h}'\nabla \cdot \widehat{\vec{u}h}$$

**Feedback term:** FMSE anom * Diabatic term anom

Positive Feedback: Process increases FMSE of already moist region
Negative Feedback: Process decreases FMSE of moist region

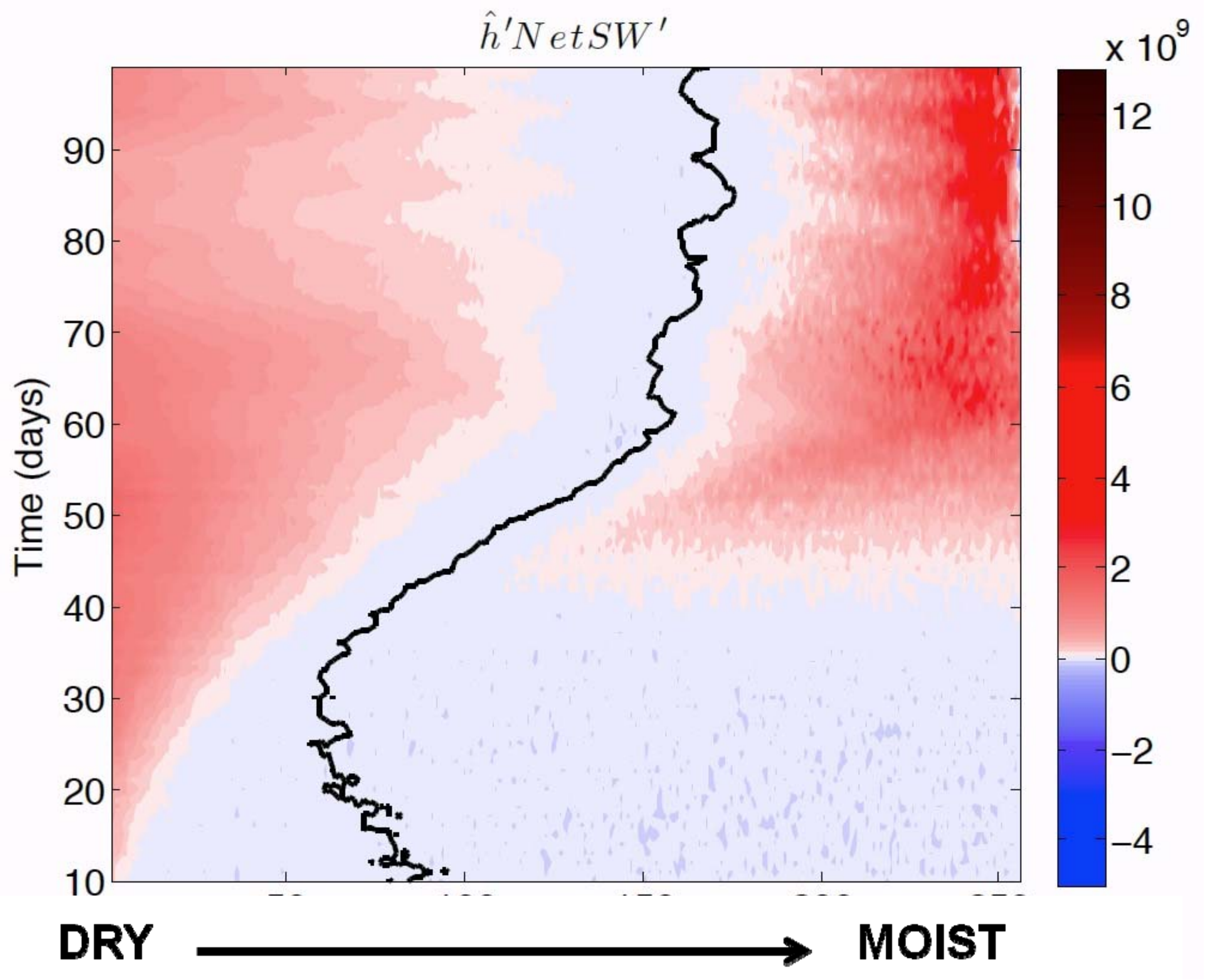
# Total Diabatic Feedback Term



**DRY** → **MOIST**

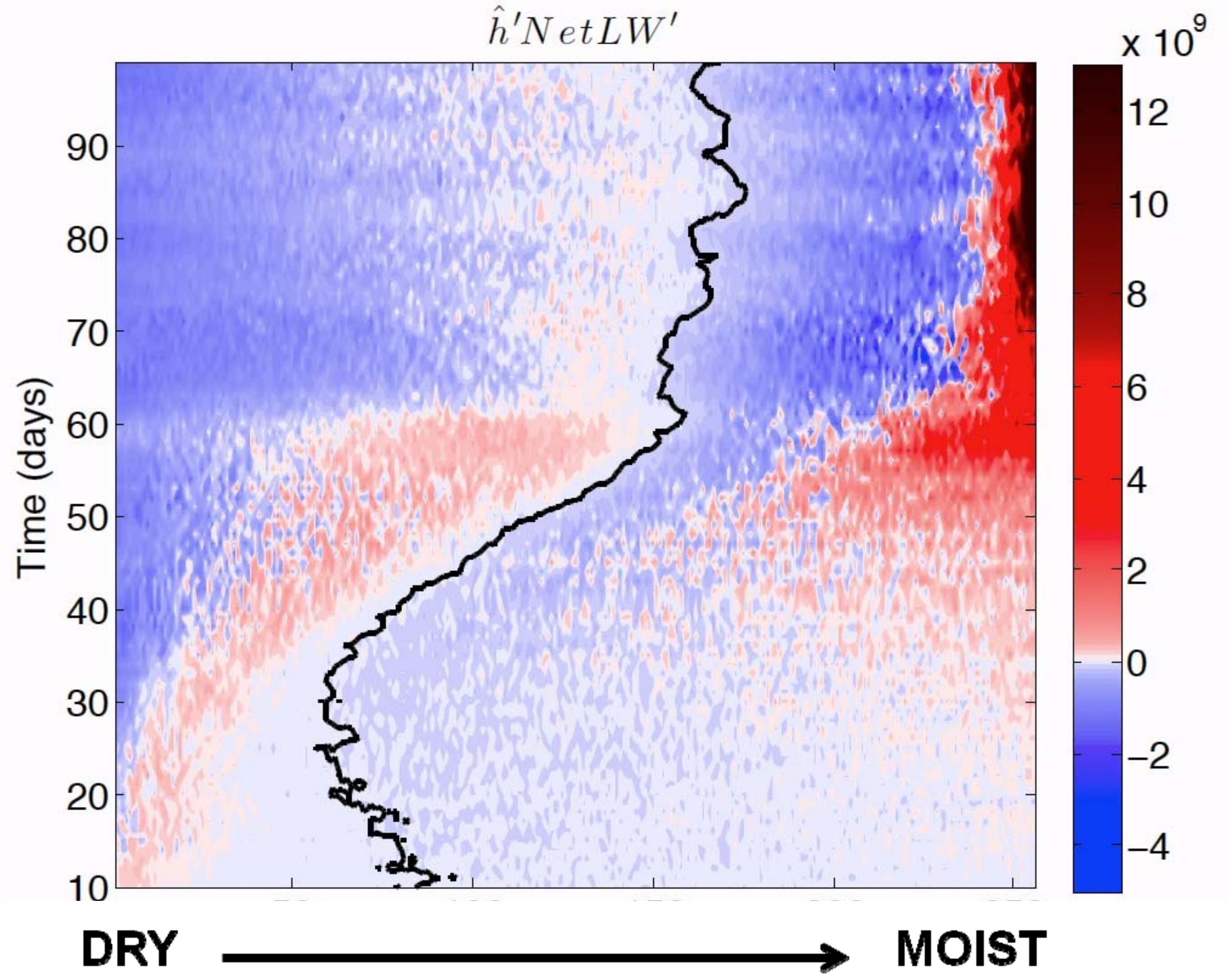
$$\frac{1}{2}\frac{\partial \hat{h}'^2}{\partial t} = \boxed{\hat{h}'\mathrm{LHF}' + \hat{h}'\mathrm{SHF}' + \hat{h}'\mathrm{NetSW}' + \hat{h}'\mathrm{NetLW}'} - \hat{h}'\nabla \cdot \widehat{\vec{u}h}$$

# Column Shortwave Flux Convergence



$$\frac{1}{2}\frac{\partial \hat{h}'^2}{\partial t} = \hat{h}'\mathrm{LHF}' + \hat{h}'\mathrm{SHF}' + \boxed{\hat{h}'\mathrm{NetSW}'} + \hat{h}'\mathrm{NetLW}' - \hat{h}'\nabla\cdot\widehat{\vec{u}h}$$

# Column Longwave Flux Convergence



$$\frac{1}{2}\frac{\partial \hat{h}'^2}{\partial t} = \hat{h}'\mathrm{LHF}' + \hat{h}'\mathrm{SHF}' + \hat{h}'\mathrm{NetSW}' + \boxed{\hat{h}'\mathrm{NetLW}'} - \hat{h}'\nabla\cdot\widehat{\vec{u}h}$$

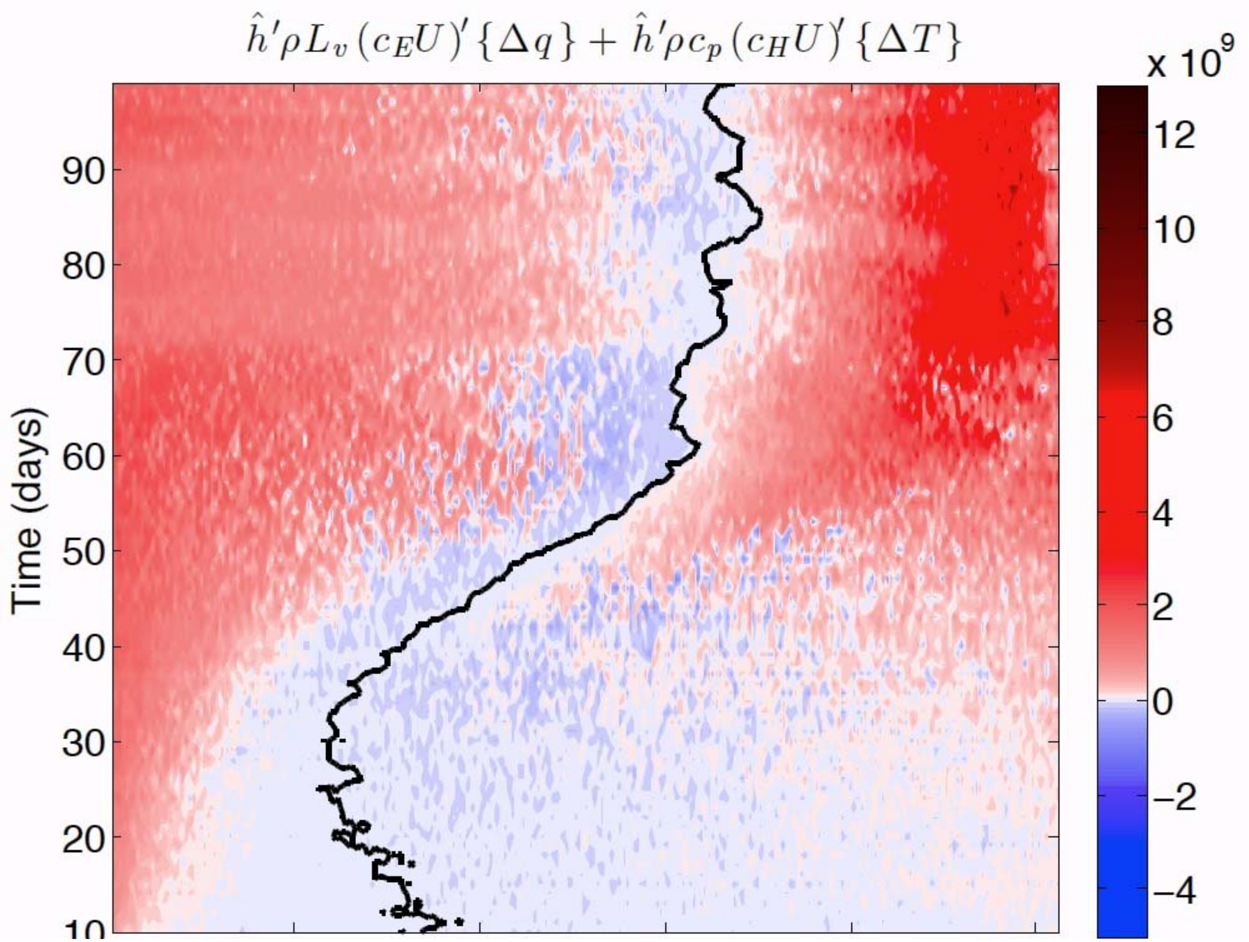
# Surface Enthalpy Flux Partitioning

$$\mathrm{LHF} = \rho c_E L_v U \left(q^*_{T_s} - q_v\right)$$

$$\mathrm{SHF} = \rho c_H c_p U \left(T_s - T_a\right)$$

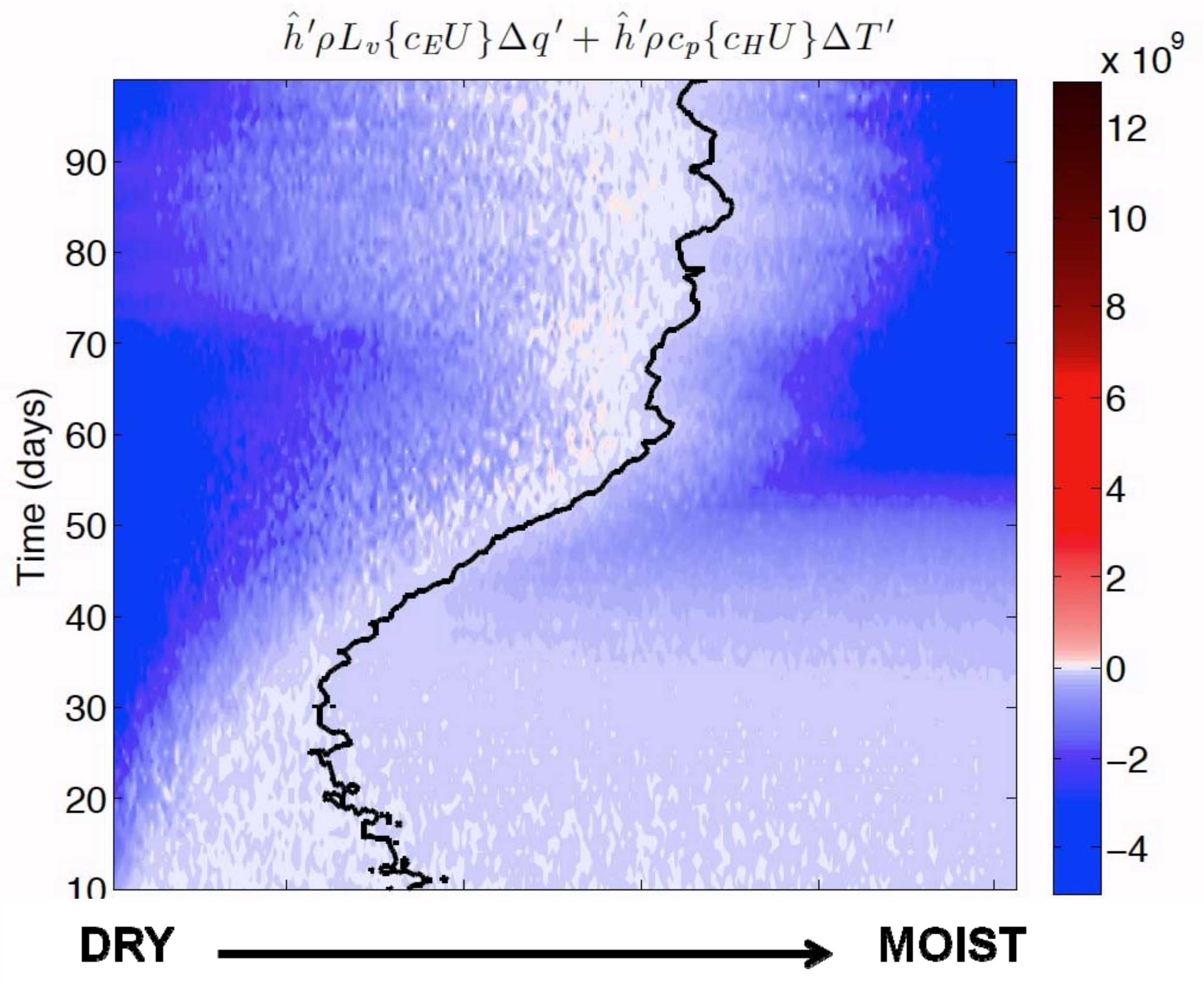
- Partition surface enthalpy flux anomalies into
  - part due to U’
  - part due to Δq’ or ΔT’
  - part due to U’Δq’ or U’ΔT’

# Surface Flux – Wind Feedback Term



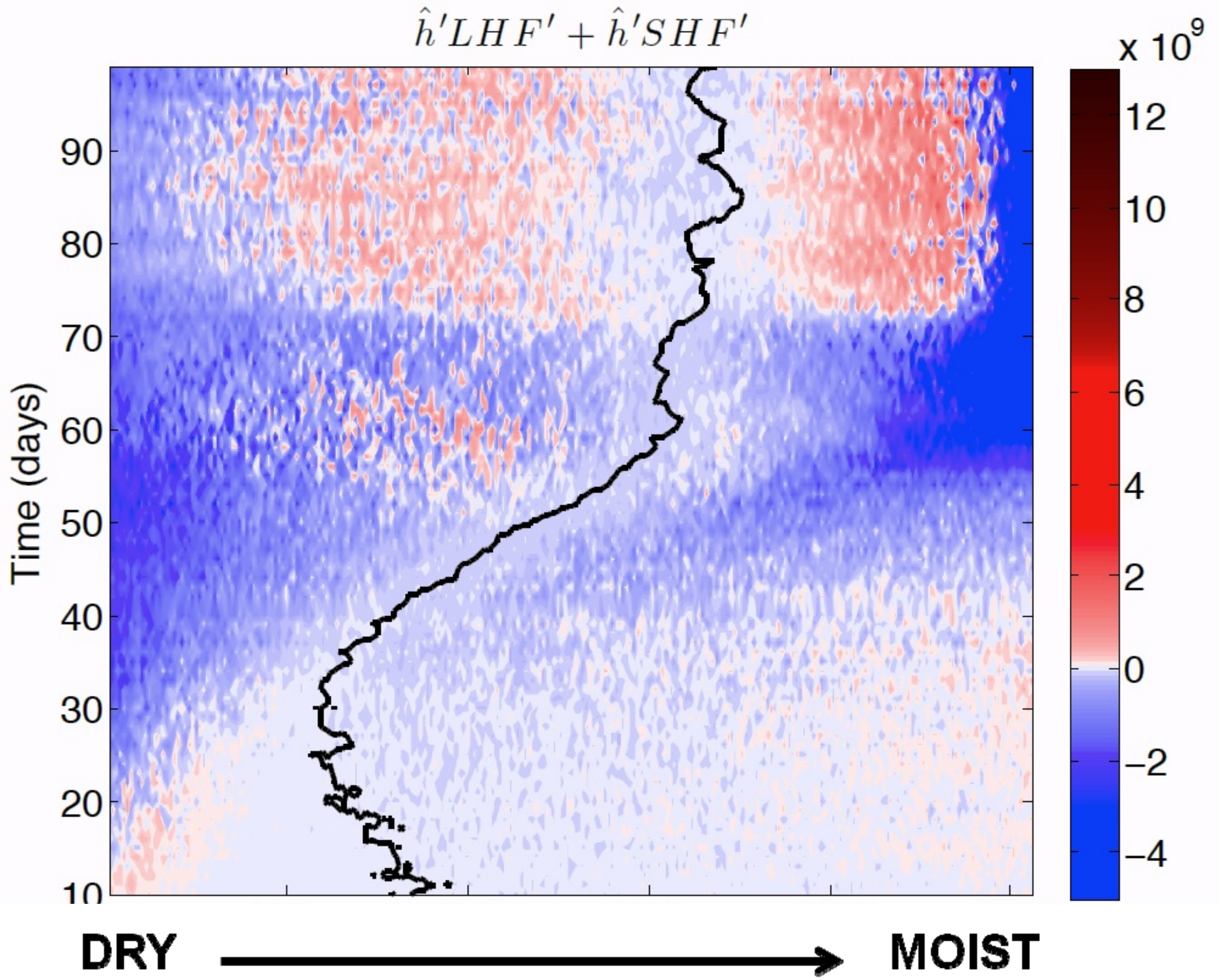
**DRY** → **MOIST**

# Surface Flux –Air-Sea Disequilibrium Feedback Term



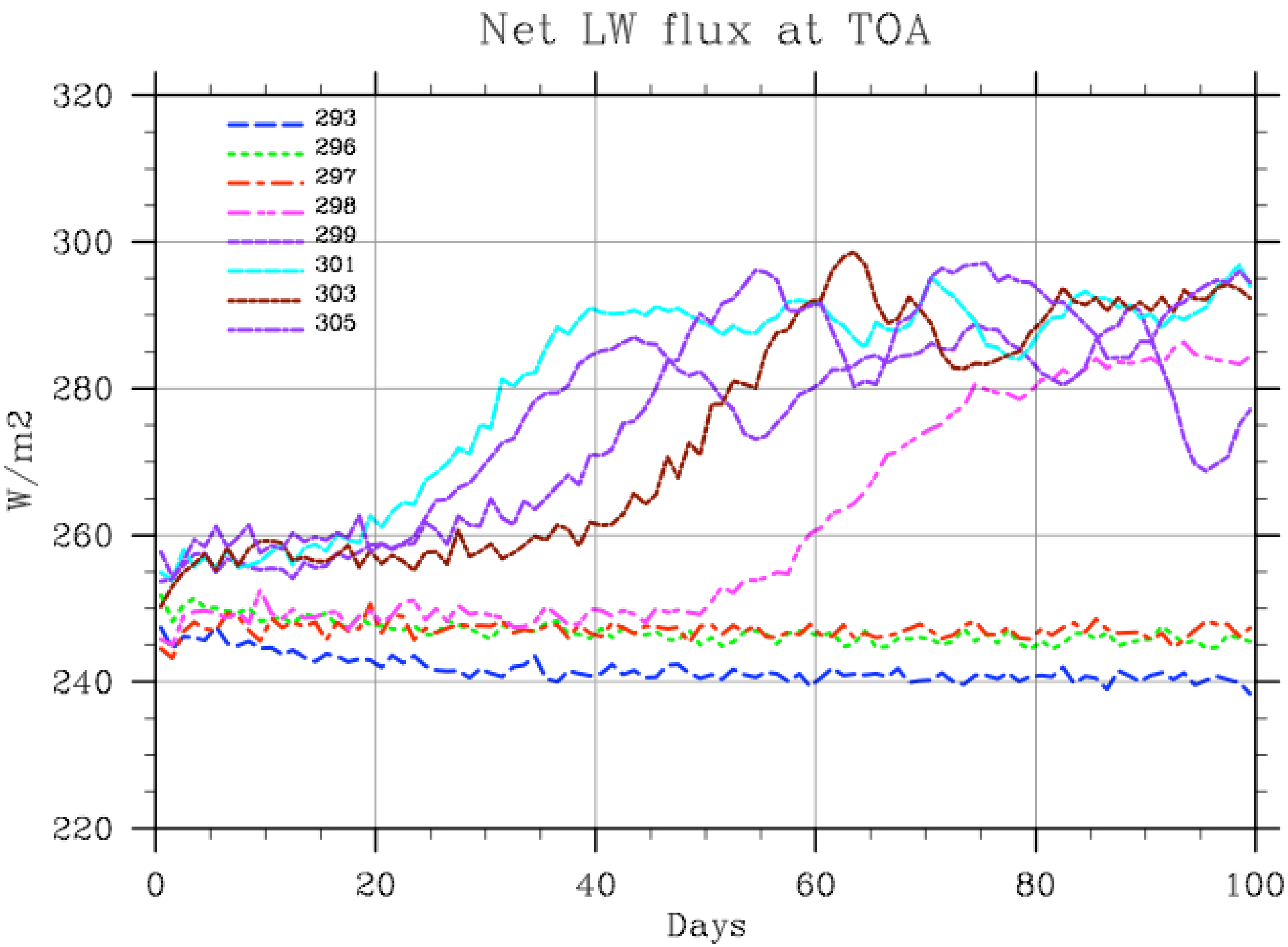
$$\hat{h}'\rho L_v\{c_E U\}\Delta q' + \hat{h}'\rho c_p\{c_H U\}\Delta T'$$

# Total Surface Flux Feedback Term



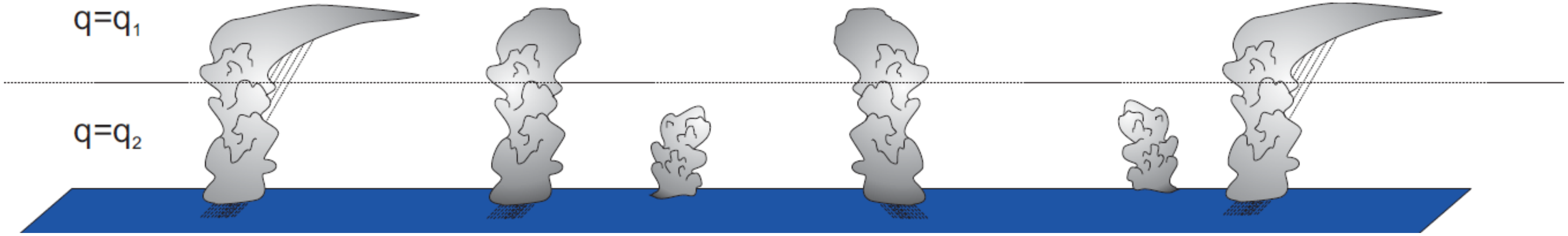
$$\frac{1}{2}\frac{\partial \hat{h}'^2}{\partial t} = \boxed{\hat{h}'\mathrm{LHF}' + \hat{h}'\mathrm{SHF}'} + \hat{h}'\mathrm{NetSW}' + \hat{h}'\mathrm{NetLW}' - \hat{h}'\nabla \cdot \widehat{\vec{u}h}$$

# **Self-Aggregation is Temperature-Dependent** (Nolan et al., 2007;  Emanuel and Khairoutdinov, in preparation, 2013)



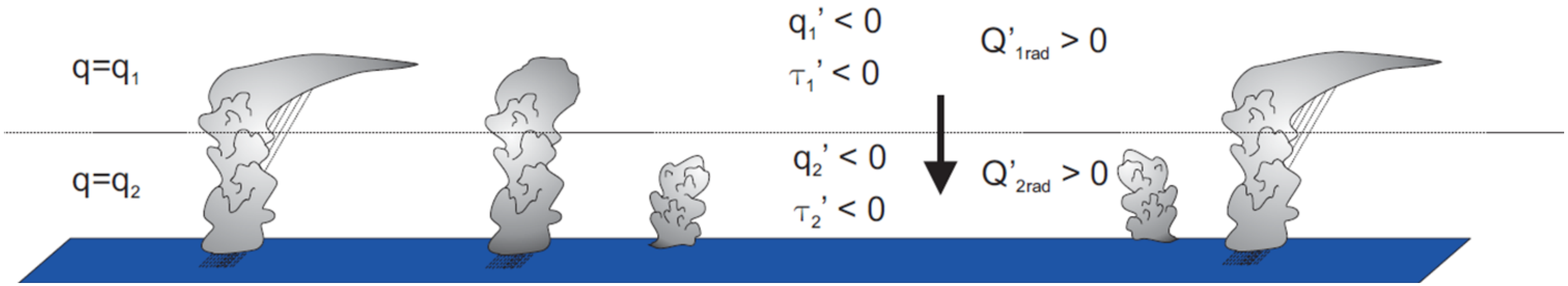
# Interpretation

## Ordinary Radiative-Convective Equilibrium

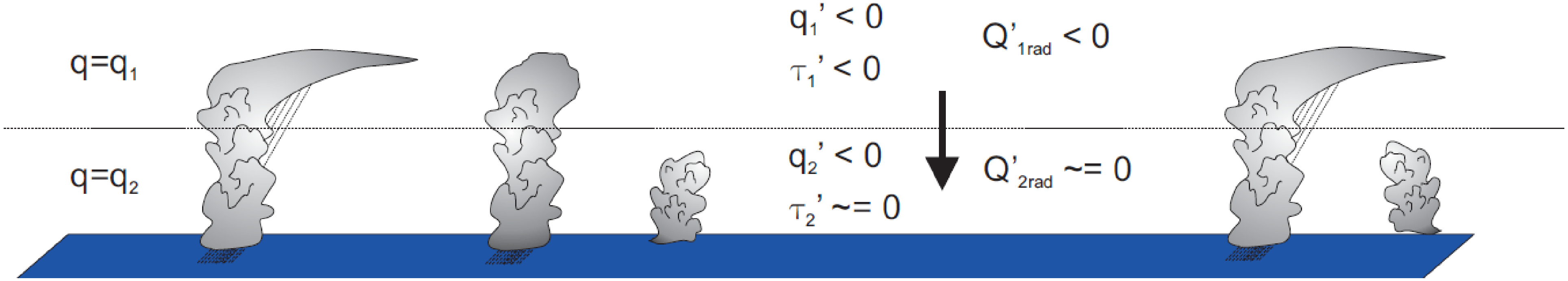


## Introduce local downward vertical velocity

**Low SST:**
Little effect on shortwave radiative heating
Reduction of longwave radiative cooling throughout column
Net positive perturbation radiative heating
Large scale ascent: Negative feedback



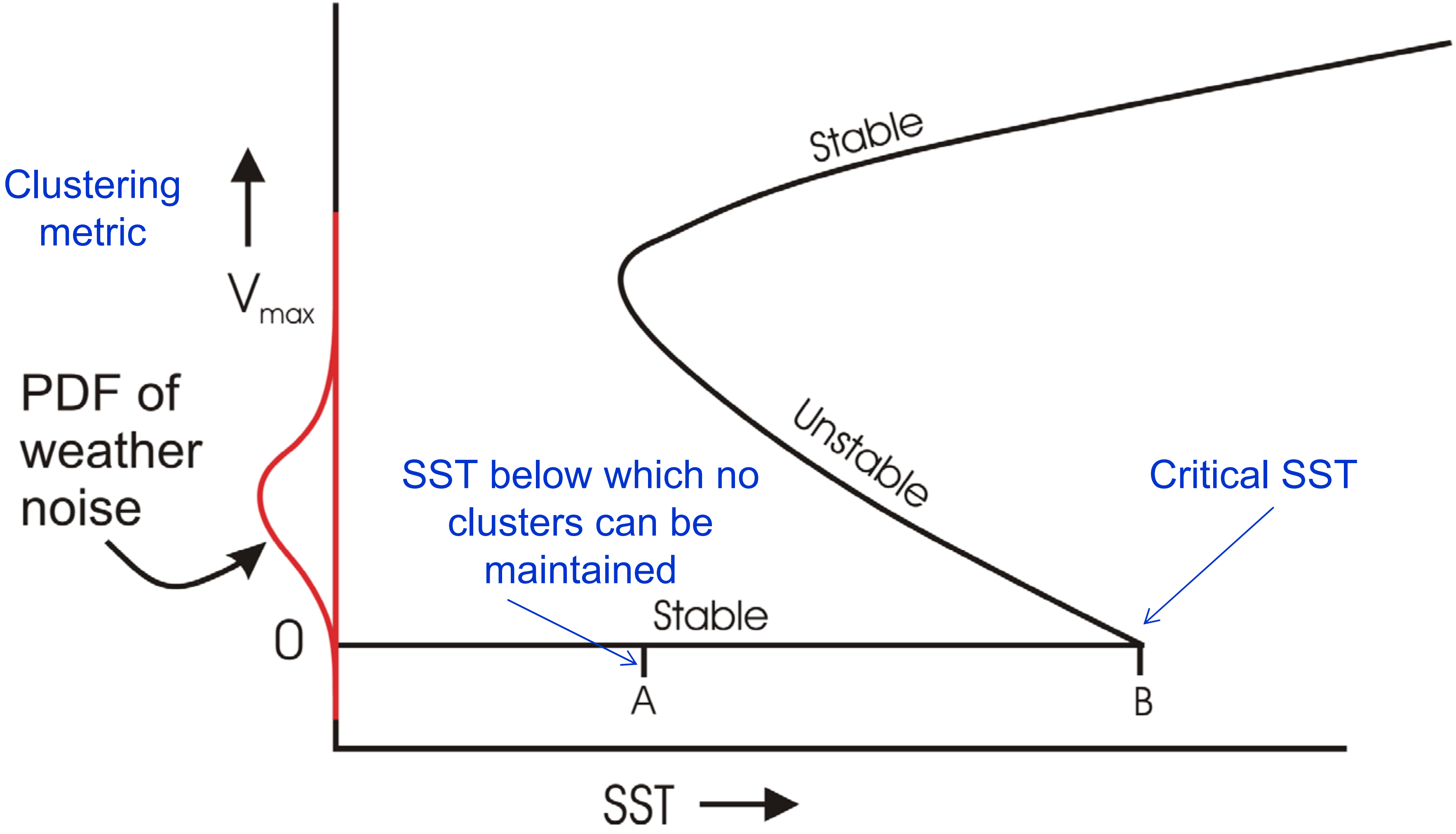
**High SST:**
Strong negative perturbations of shortwave heating
Reduction of longwave radiative cooling in upper troposphere
Increased longwave cooling of lower troposphere
Net negative perturbation radiative heating
Large scale descent: **Positive feedback**



# Note:

**Once cluster forms, it is strongly maintained by intense negative OLR anomaly associated with central dense overcast. But cloud feedbacks are NOT important in instigating the instability! This leads to strong hysteresis in the radiative-convective system**

# Hypothesized Subcritical Bifurcation



Clustering metric

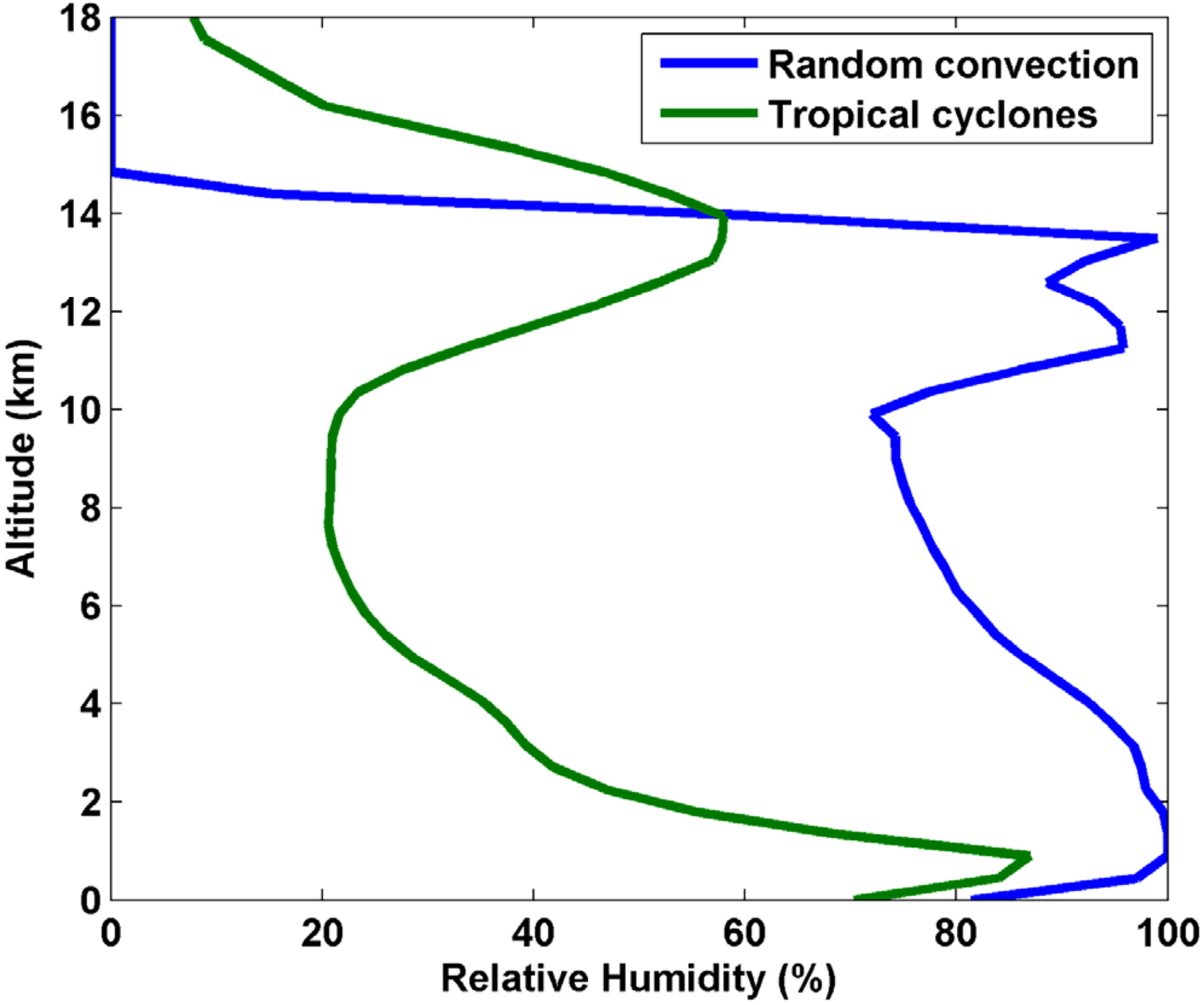
SST below which no clusters can be maintained

Critical SST

from Emanuel and Nolan (2004)

# Consequences

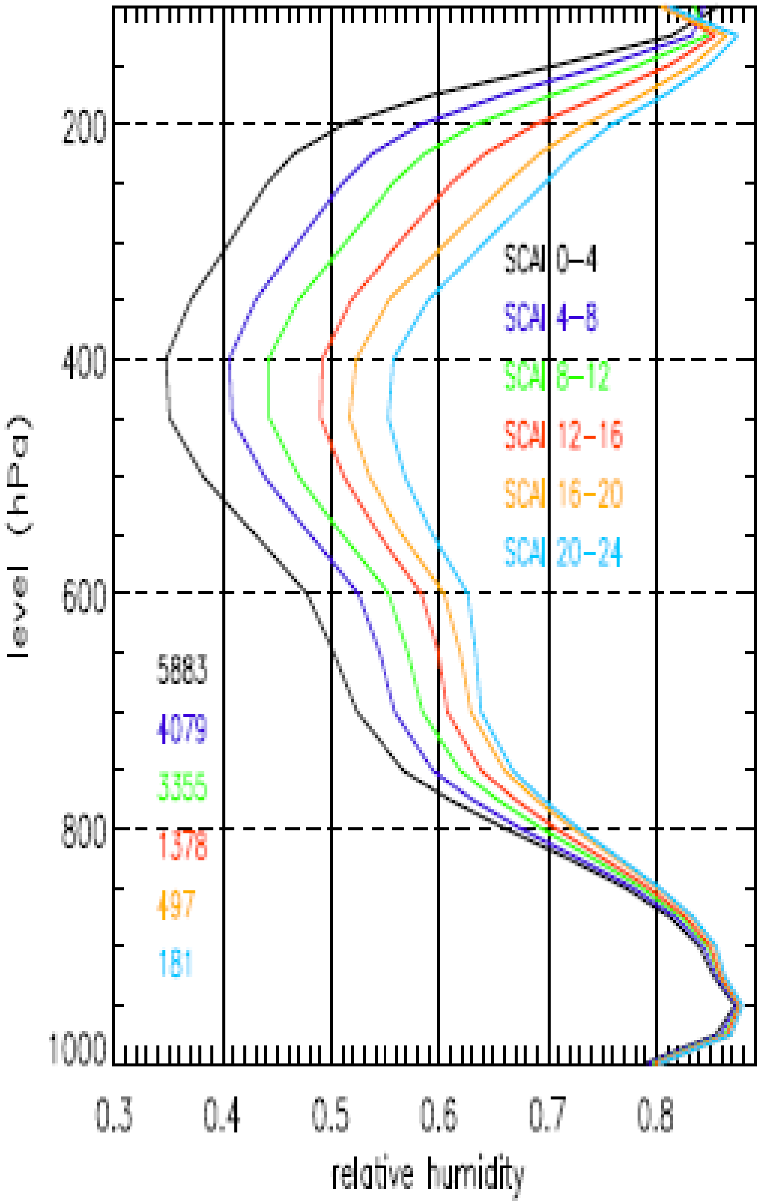
# Nolan et al., QJRMS, 2007



Variation of tropical relative humidity profiles with a Simple Convective Aggregation Index (SCAI).

*Courtesy Isabelle Tobin, Sandrine Bony, and Remy Roca*

Tobin, Bony, and Roca, *J. Climate*, 2012



# Hypothesis

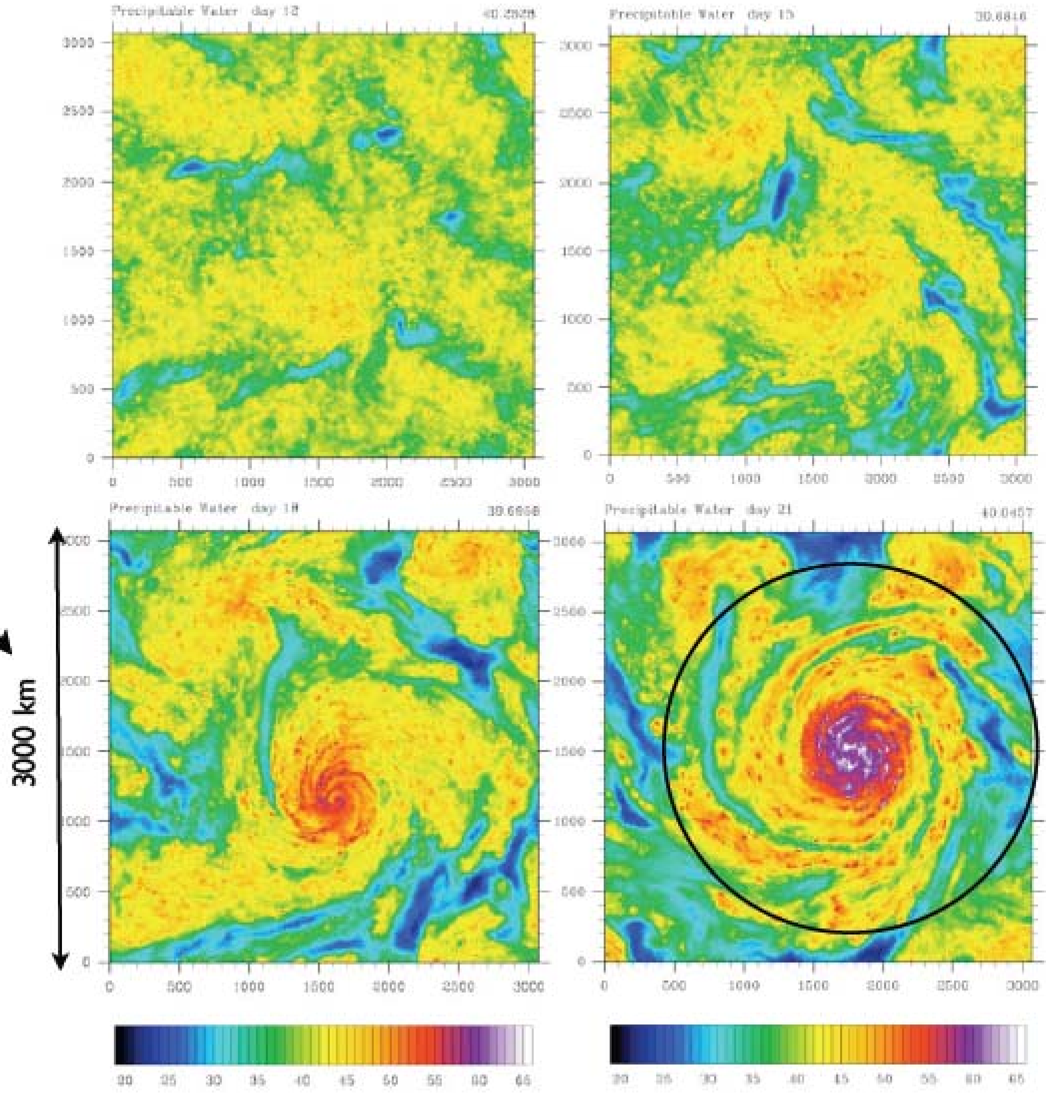
- At high temperature, convection self-aggregates
- →Horizontally averaged humidity drops dramatically
- →Reduced greenhouse effect cools system
- →Convection disaggregates
- →Humidity increases, system warms
- →System wants to be near phase transition to aggregated state

# Recipe for Self-Organized Criticality

## (First proposed by David Neelin, but by different mechanism)

- System should reside near critical threshold for self-aggregation
- Convective cluster size should follow power law distribution

# Self-Aggregation on an f-plane



Expensive!

TC size:

$$R = \frac{V_{PI}}{f}$$

Emanuel's potential Intensity

$$V_{PI} \approx 70 m/s$$

Coriolis parameter(20$^{O}$N)

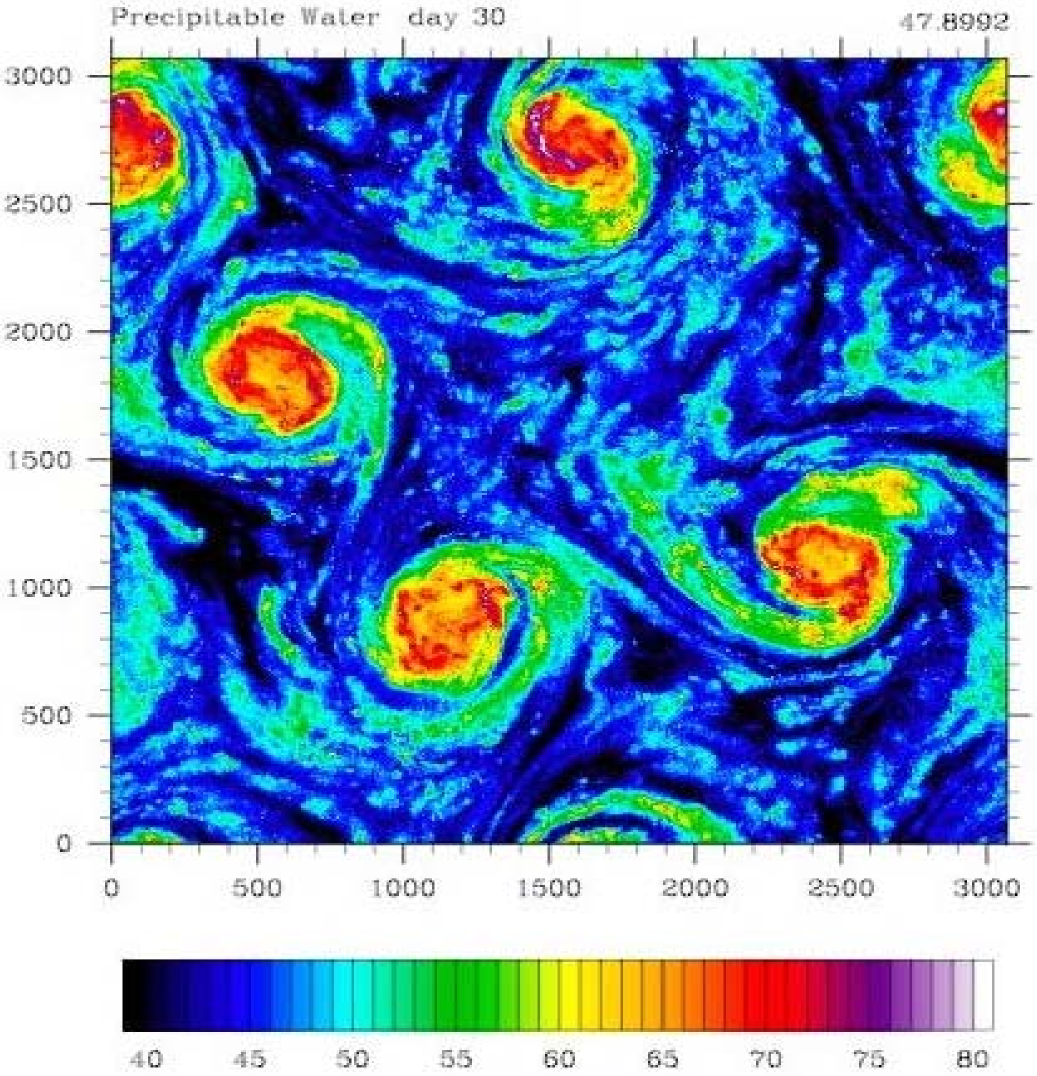
$$f \approx 5 \times 10^{-5} s^{-1}$$

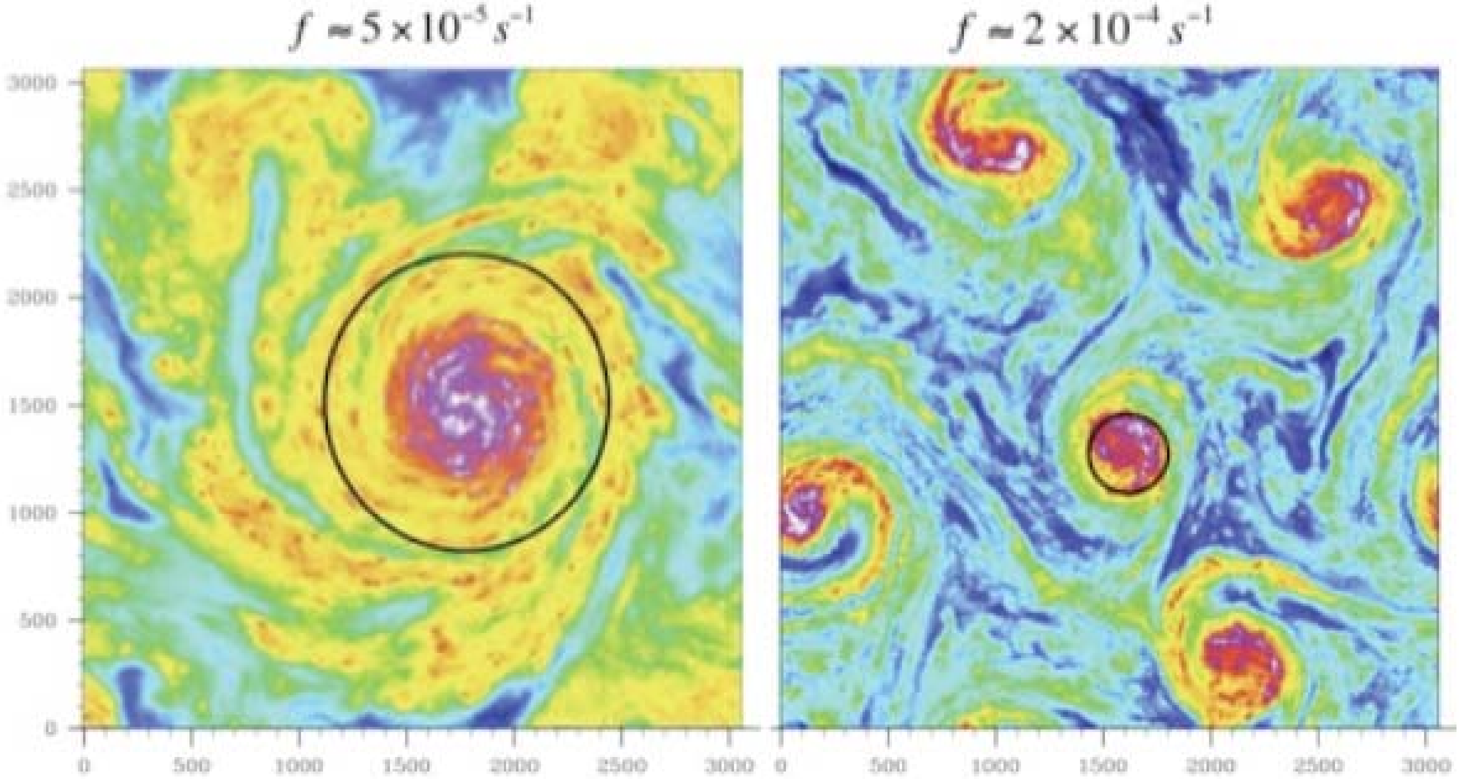
$$R = 1400 km$$

(Khairoutdinov 2009)

Vincent Van Gogh: Starry Night

Distance between vortex centers scales as $V_{pot}/f$




$f \approx 5 \times 10^{-5}\,s^{-1}$

$f \approx 2 \times 10^{-4}\,s^{-1}$

**Figure 1** Simulated tropical cyclones for two different values of Coriolis parameter in otherwise identical RCE simulations. The superimposed circles show the characteristic sizes of the cyclones as estimated from the expression (2).

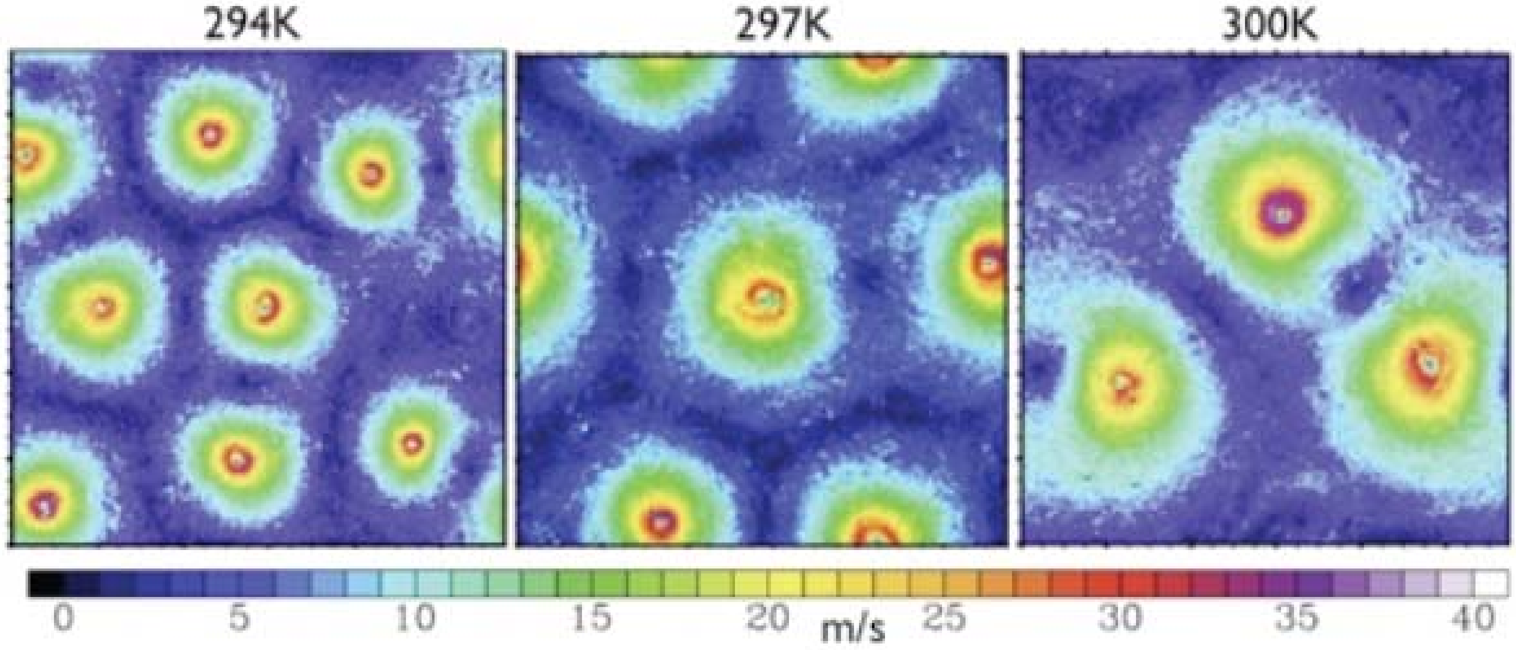


**Figure 2** Snapshots of near-surface wind (in m/s) in RCE with rotation for three different values of the SST.

# TC-World Scaling

- Frequency ~ $\frac{f^2}{V_{pot}^2}$
- Intensity ~ $V_{pot}$
- Power Dissipation ~ $V_{pot} f^2$ (rises quickly as SST increases and expands poleward)

# Summary

- Radiative-Convective Equilibrium remains an interesting problem in climate science
- At high temperature, RCE is unstable, owing to the particular dependencies of convection and radiation on atmospheric water vapor and clouds
- Aggregation of convection may have profound effects on climate
- Physics of aggregation may not operate well, if at all, in today’s GCMs