Low Clouds in The Hadley Circulation

Figure copied from Albrecht (BAMS, 1995)

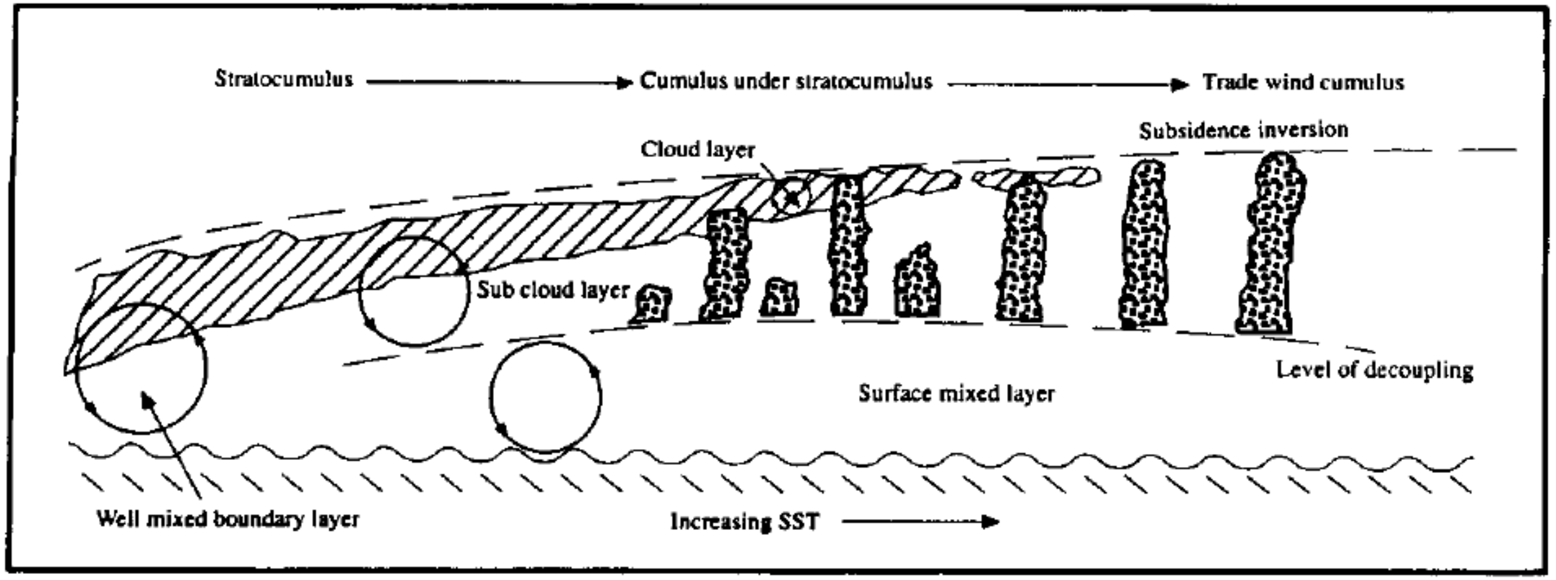
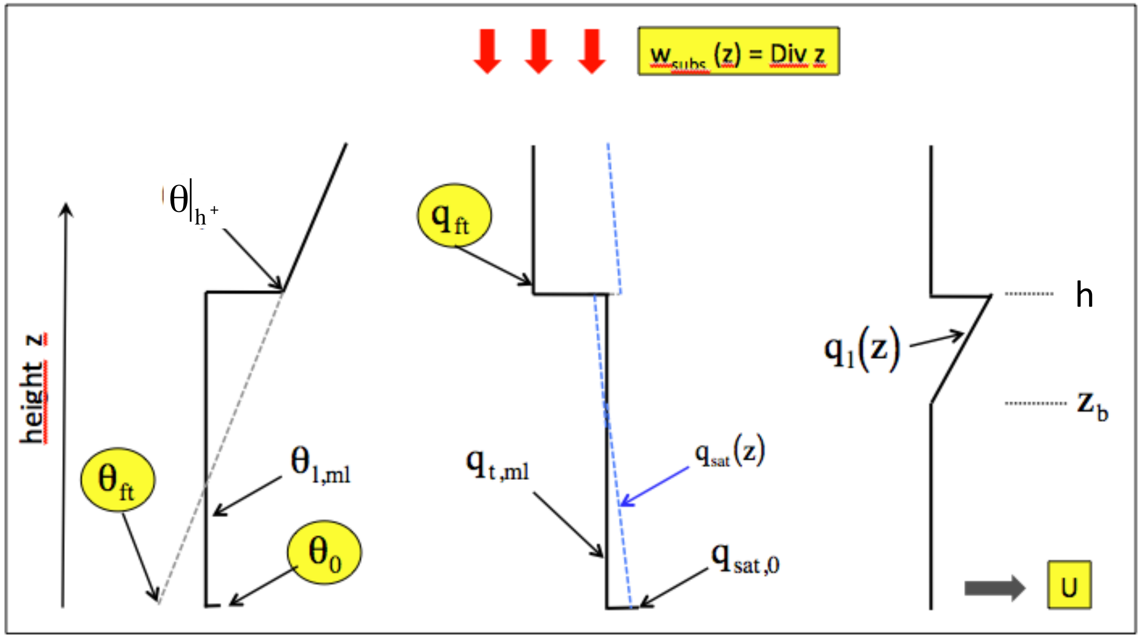
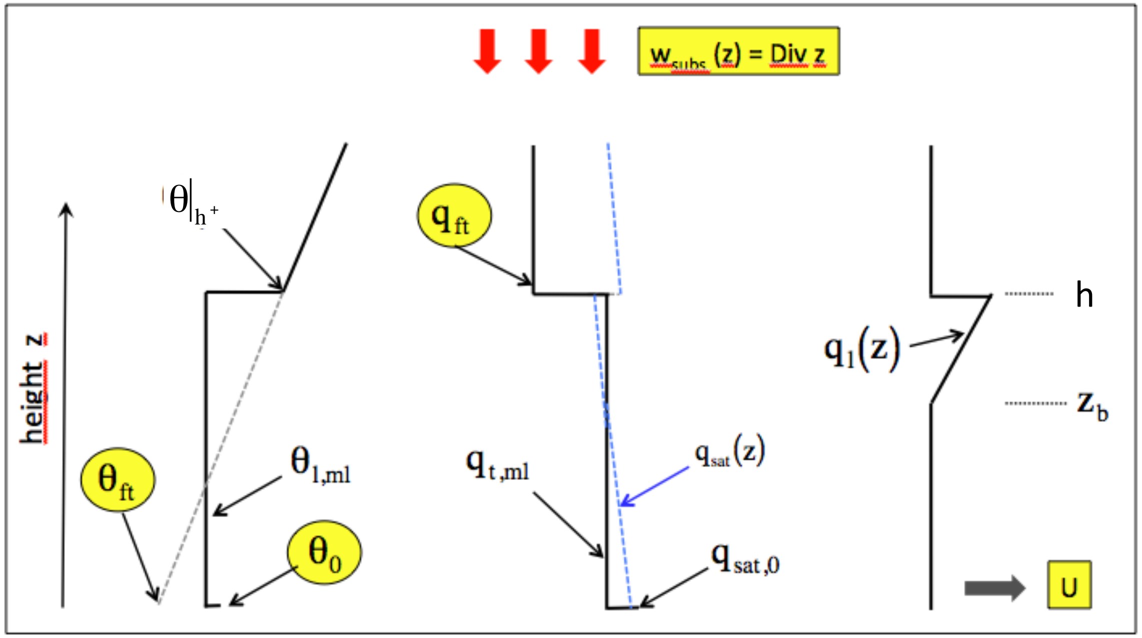


FIG. 4. A schematic of the transition from stratocumulus to trade wind cumulus.

Set-up of the mixed layer model

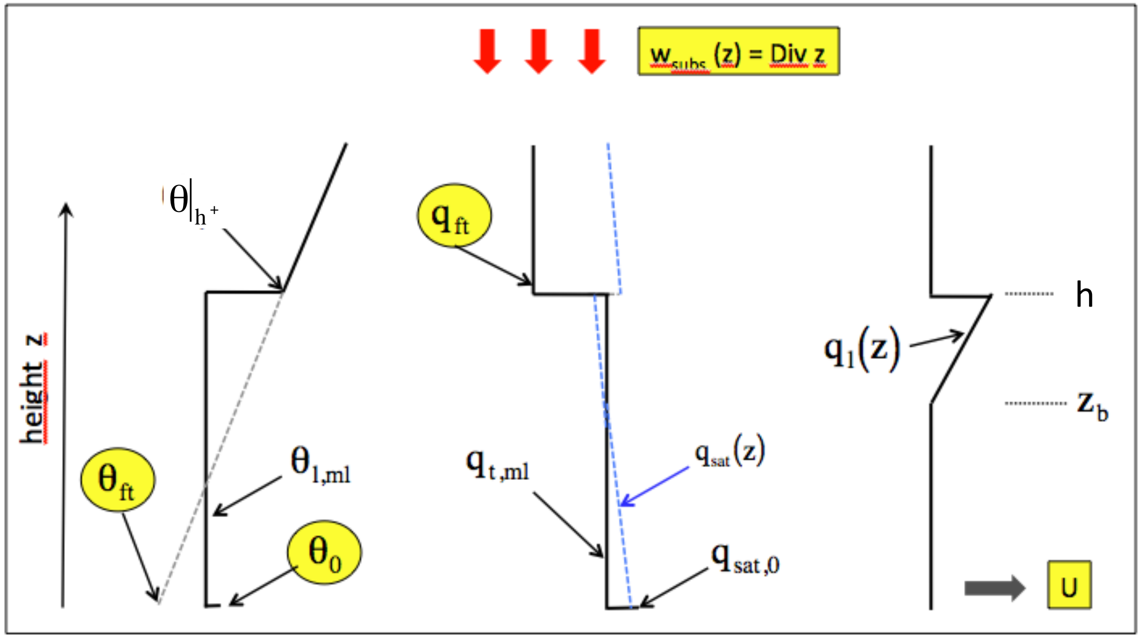


Budget equations for the stratocumulus-topped boundary layer



Mass $\quad \dfrac{\partial h}{\partial t} = w_e - Dh \quad , \quad \overline{w}_{subs}\Big|_h = -Dh$

Budget equations for the stratocumulus-topped boundary layer



Mass $$\frac{\partial h}{\partial t} = w_e - Dh$$

Heat $$h\frac{\partial \theta_{l,ml}}{\partial t} = C_d U\left(\theta_0 - \theta_{l,ml}\right) + w_e\left(\theta|_{h^+} - \theta_{l,ml}\right) - \Delta S_{\theta_l}$$

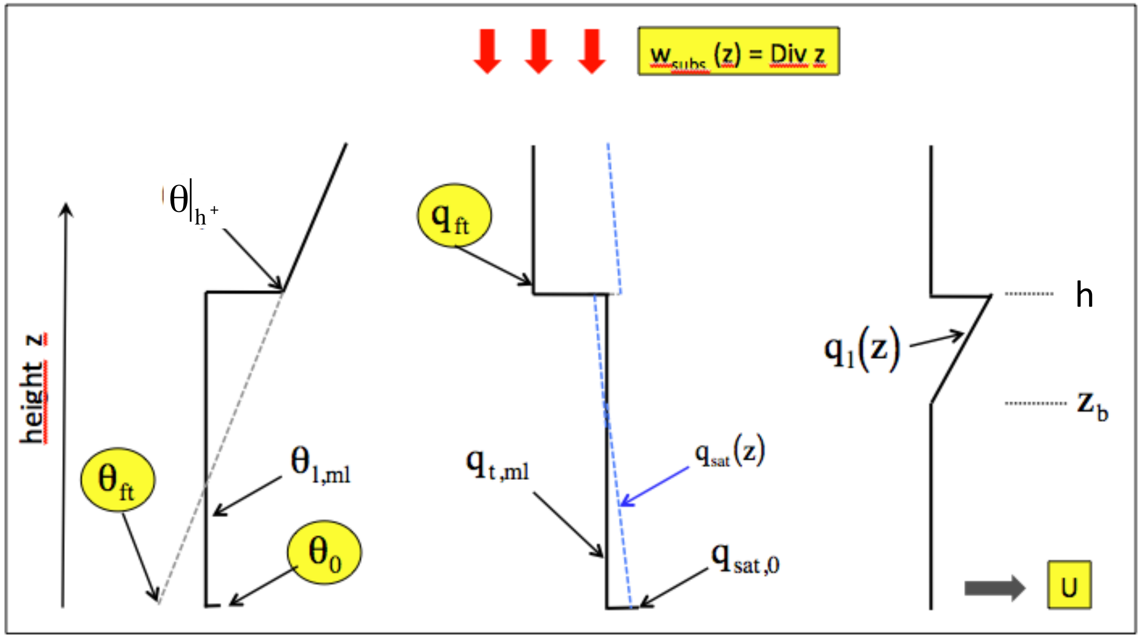
surface flux
C_d = 0.001

entrainment flux

radiative flux divergence
between surface and cloud top

(*) horizontal advection may be plugged in source term $\Delta S_{\theta l}$

Budget equations for the stratocumulus-topped boundary layer



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Water $$h\frac{\partial q_{t,ml}}{\partial t} = C_d U\left(q_{sat,0} - q_{t,ml}\right) + w_e\left(q_{ft} - q_{t,ml}\right) - \Delta S_{q_t}$$

surface flux — entrainment flux — drizzle (neglected today)

Equilibrium Solutions Using A Simple Entrainment Parameterization

$$\theta_{l,ml} = \theta_0 + \frac{(\eta - 1)\Delta F}{C_d U} \qquad \text{with } w_e = \eta \frac{\Delta F}{\Delta \overline{\theta}_l} \quad , \quad \Delta F > 0$$

Equilibrium Solutions Using A Simple Entrainment Parameterization

$$\theta_{l,ml} = \theta_0 + \frac{(\eta - 1)\Delta F}{C_d U} \qquad \text{with} \quad w_e = \eta \frac{\Delta F}{\Delta \overline{\theta}_l}$$

$$q_{t,ml} = q_{sat,0} + \frac{w_e (q_{ft} - q_{sat,0})}{w_e + C_d U}$$

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example η=1, θ_{ft}=θ_0

$$h^2 + \frac{h}{\Gamma_\theta}\left[\theta_{ft} - \theta_0 + \frac{(1-\eta)\Delta F}{C_d U}\right] - \frac{\eta}{D\Gamma_\theta}\Delta F = 0 \quad \Longrightarrow \quad \boxed{h = \sqrt{\frac{\Delta F}{D\Gamma_\theta}}}$$

High cloud-top h if

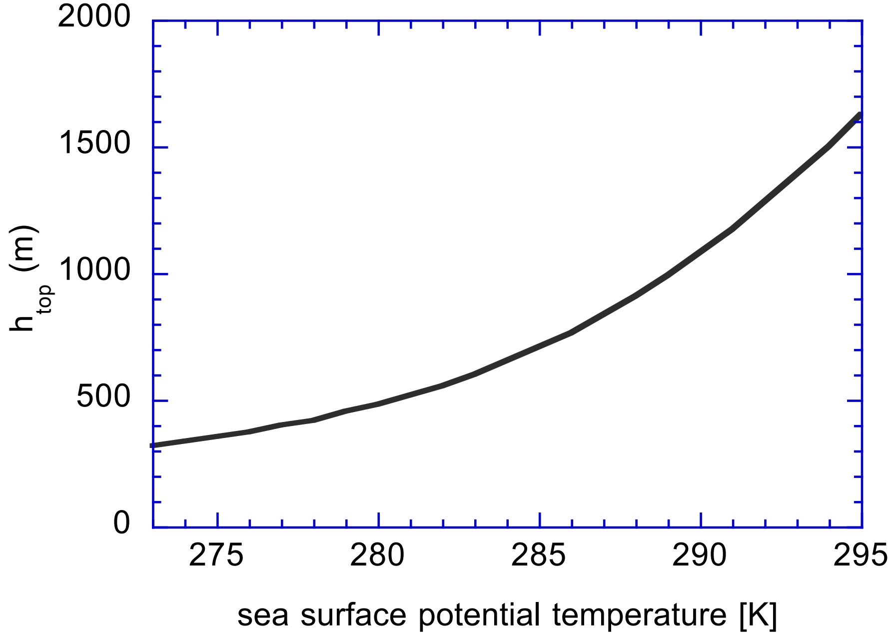
Weak large-scale divergence D

Strong cloud radiative cooling ΔF

low Γ_θ: cold free troposphere (with respect to the sea surface temperature)

Equilibrium Solutions For The Mixed-Layer Height

$$h^2 + \frac{h}{\Gamma_\theta}\left[\theta_{ft} - \theta_0 + \frac{(1-\eta)\Delta F}{C_d U}\right] - \frac{\eta}{D\Gamma_\theta}\Delta F = 0$$



$D = 5.10^{-6}\ s^{-1}$

$U = 10\ ms^{-1}$

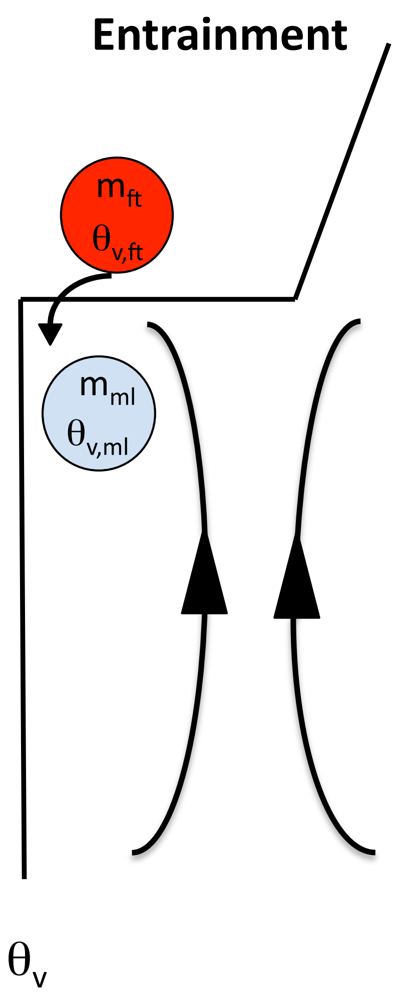
$\Delta F = 0.035\ mKs^{-1}$

$\theta_{ft} = 288\ K$

$\Gamma_\theta = 6\ K\ km^{-1}$

$\eta = 0.8$

Mixing across the inversion: dry case



The two parcels mix

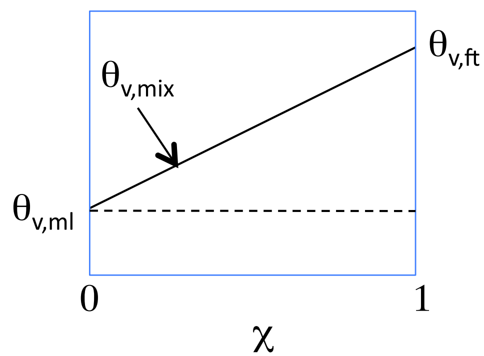
$m_{ml}+m_{ft}$

$\theta_{v,mix}$

$$(m_{ml}+m_{ft})\theta_{v,mix} = m_{ml}\theta_{v,ml} + m_{ft}\theta_{v,ft}$$

$$\chi = \frac{m_{ft}}{m_{ml}+m_{ft}}$$

$$\theta_{v,mix} = (1-\chi)\theta_{v,ml} + \chi\theta_{v,ft}$$



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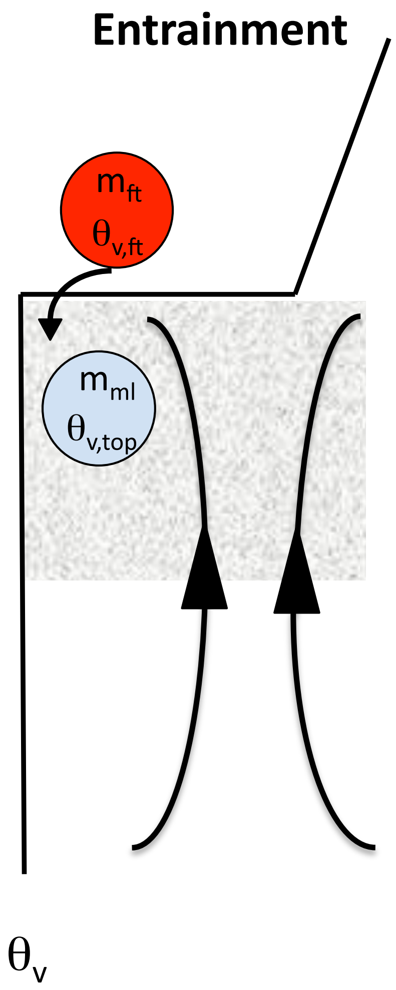
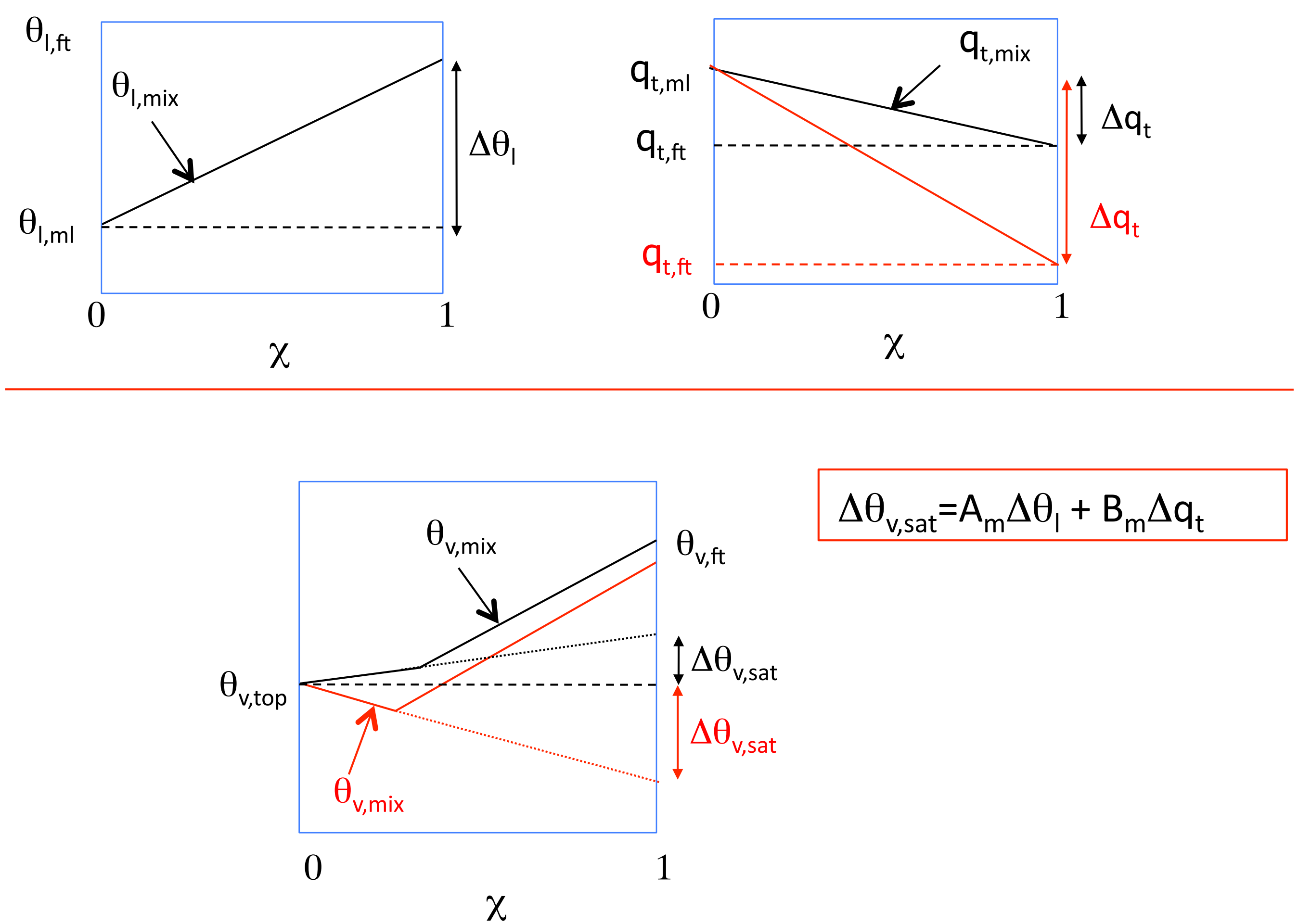




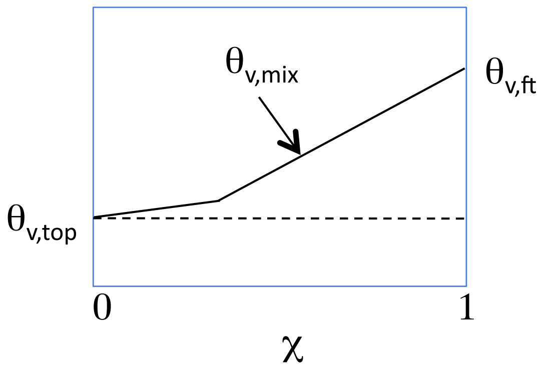
$$\Delta\theta_{v,sat} = A_m \Delta\theta_l + B_m \Delta q_t$$

Stratocumulus entrainment parameterization

$$w_*^3 = 2.5\int_0^h \frac{g}{\theta_0}\overline{w'\theta_v'}dz \qquad Ri_{w_*} = \frac{gh}{\theta_0}\frac{\overline{\Delta\theta_v}}{w_*^2} \qquad \frac{w_e}{w_*} = \frac{A}{Ri_{w_*}} \quad \Leftrightarrow \quad w_e = A\frac{w_*^3}{\frac{gh}{\theta_0}\overline{\Delta\theta_v}}$$

Entrainment enhancement by evaporative cooling (Nicholls and Turton 1986)

$$\Delta_m = 2\int_0^1\left[\theta_{v,mix}(\chi) - \theta_{v,top}\right]d\chi$$

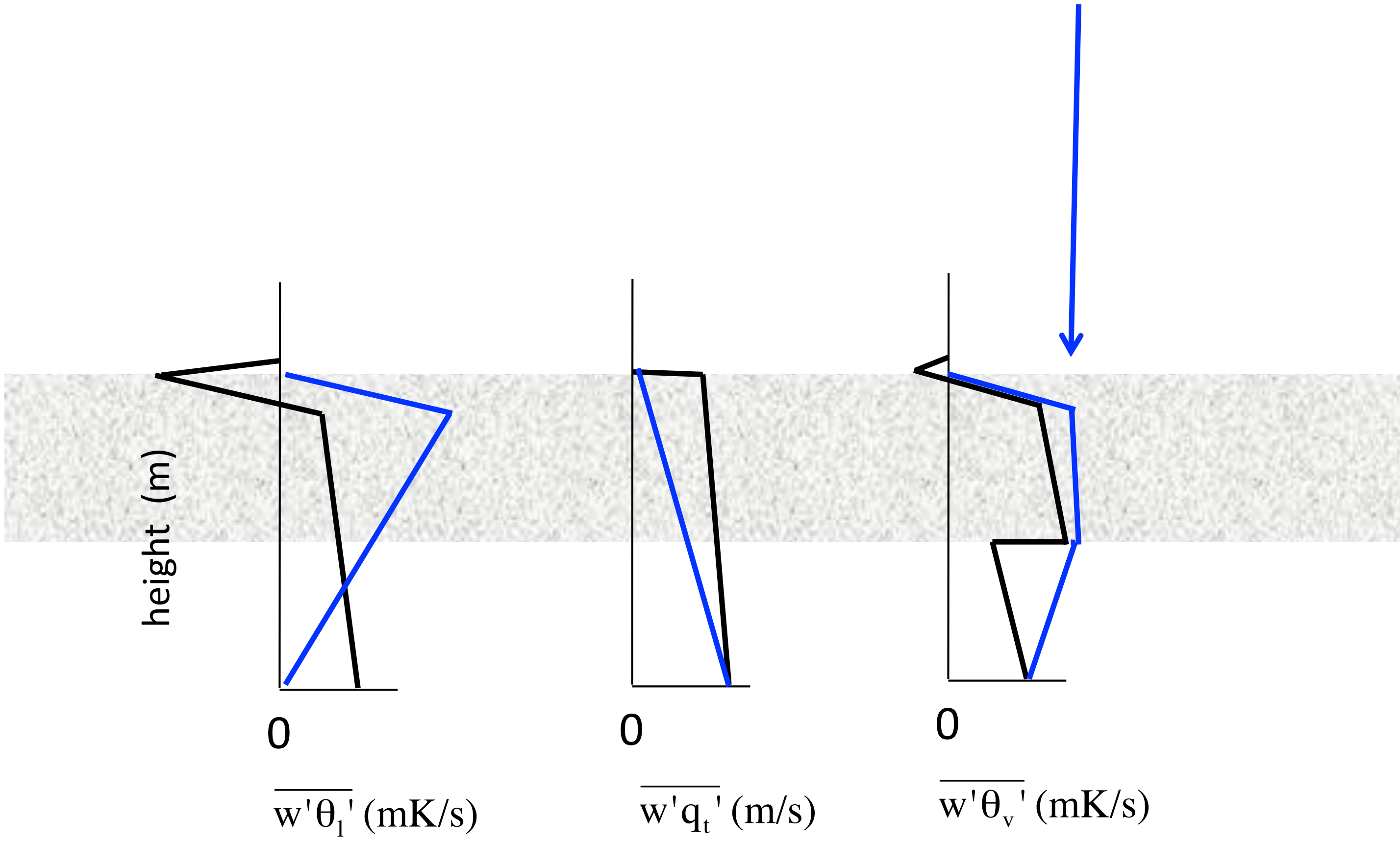


$$w_e = A_{NT}\frac{w_*^3}{\frac{gh}{\theta_0}\overline{\Delta\theta_v}}$$

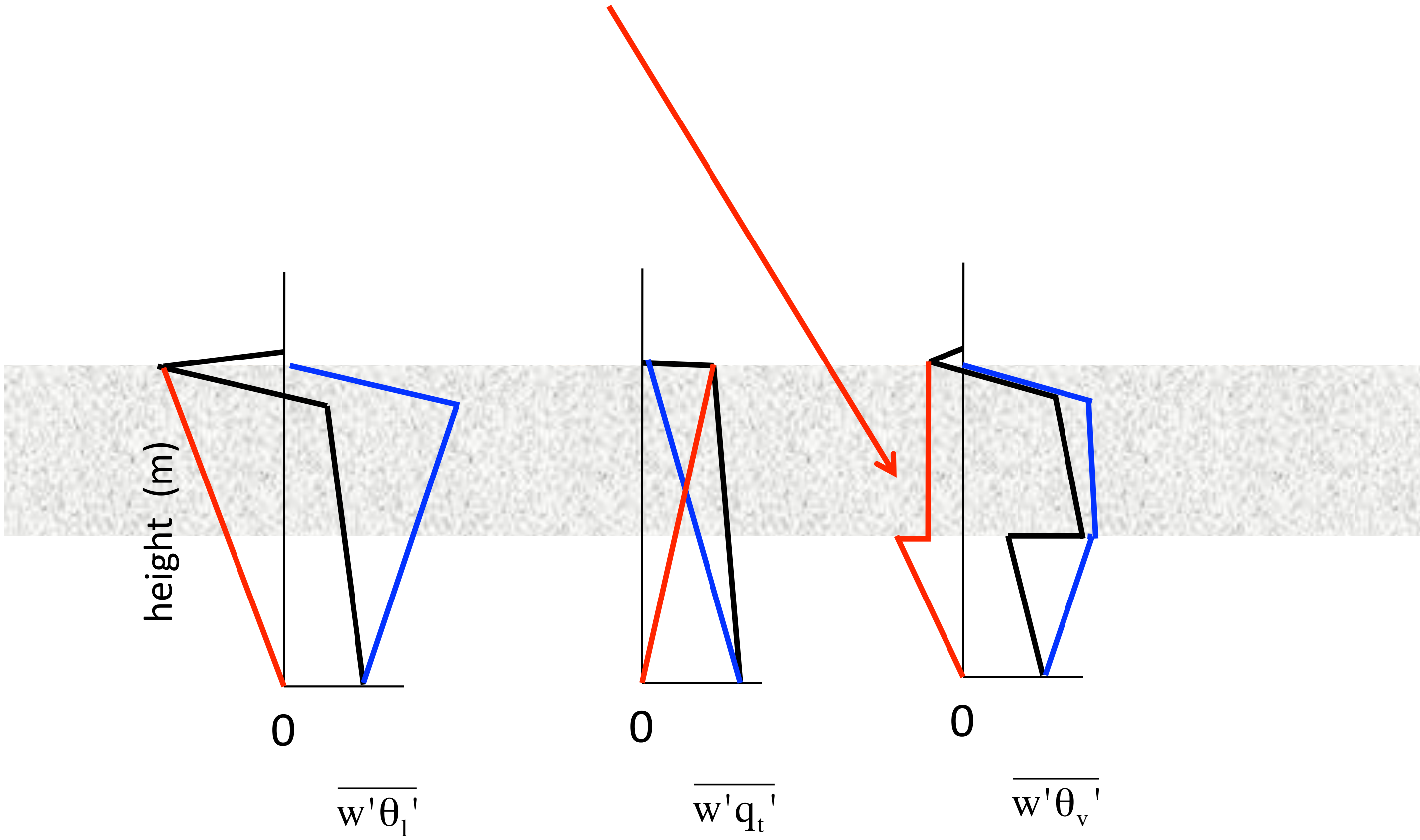
$$A_{NT} = A\left[1 + a_2\left(1 - \frac{\Delta_m}{\Delta\theta_v}\right)\right] \geq A$$

stronger evaporative cooling effect,
larger entrainment factor A

Buoyancy flux without entrainment



Buoyancy flux due to entrainment



Stratocumulus entrainment parameterization

$$w_e = A_{NT} \frac{w_*^3}{\frac{gh}{\overline{\overline{\theta_v}}} \Delta\overline{\theta_v}}$$

$$A_{NT} = A\left[1 + a_2\left(1 - \frac{\Delta_m}{\Delta\theta_v}\right)\right]$$

↓ w_* depends on w_e

$$w_e = \frac{\frac{1}{h}\int_0^h \overline{w'\theta_v'}_{\text{no entrainment}} dz}{\frac{\Delta\theta_v}{1 + a_2\left(1 - \frac{\Delta_m}{\Delta\theta_v}\right)} + f_1\Delta\theta_v + f_2\Delta\theta_{v,sat}} = \frac{\text{forcing}}{\text{measure of inversion stability}}$$

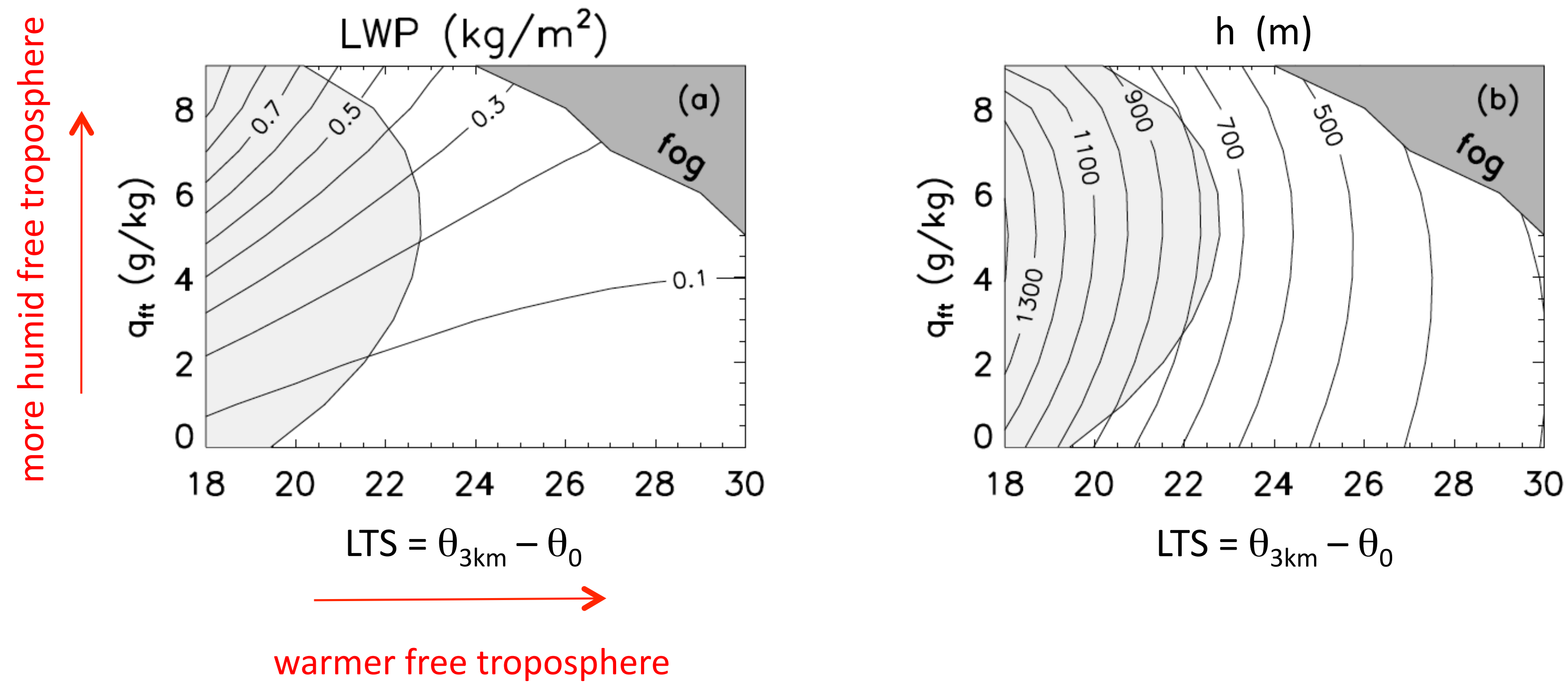
$$f_1 = \frac{1}{2}\left(\frac{z_{base}}{h}\right)^2 \quad , \quad f_2 = \frac{1}{2}\left[1 - \left(\frac{z_{base}}{h}\right)^2\right]$$

arise from w_* (vertical integral of buoyancy flux)

1. As input we need to know $\overline{w'\theta'}_0, \overline{w'q}_0, z_{base}, h, \Delta F_{rad}, \Delta\theta_l, \Delta q_t$
2. Most entrainment parameterizations have a similar form (forcing/inversion strength measure) (see Stevens, 2002)

Equilibrium solutions
(Nicholls and Turton entrainment parameterization)

Variable φ	Units	Reference value
θ_0	(K)	288.0
D	(s^{-1})	$5 \cdot 10^{-6}$
U	(ms^{-1}	10.0
ΔF	(Kms^{-1})	0.035
θ_{ft}	(K)	[285,301]
q_{ft}	($g\ kg^{-1}$)	[0,9]
Γ_θ	($K\ km^{-1}$)	6.0



Equilibrium solutions
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