

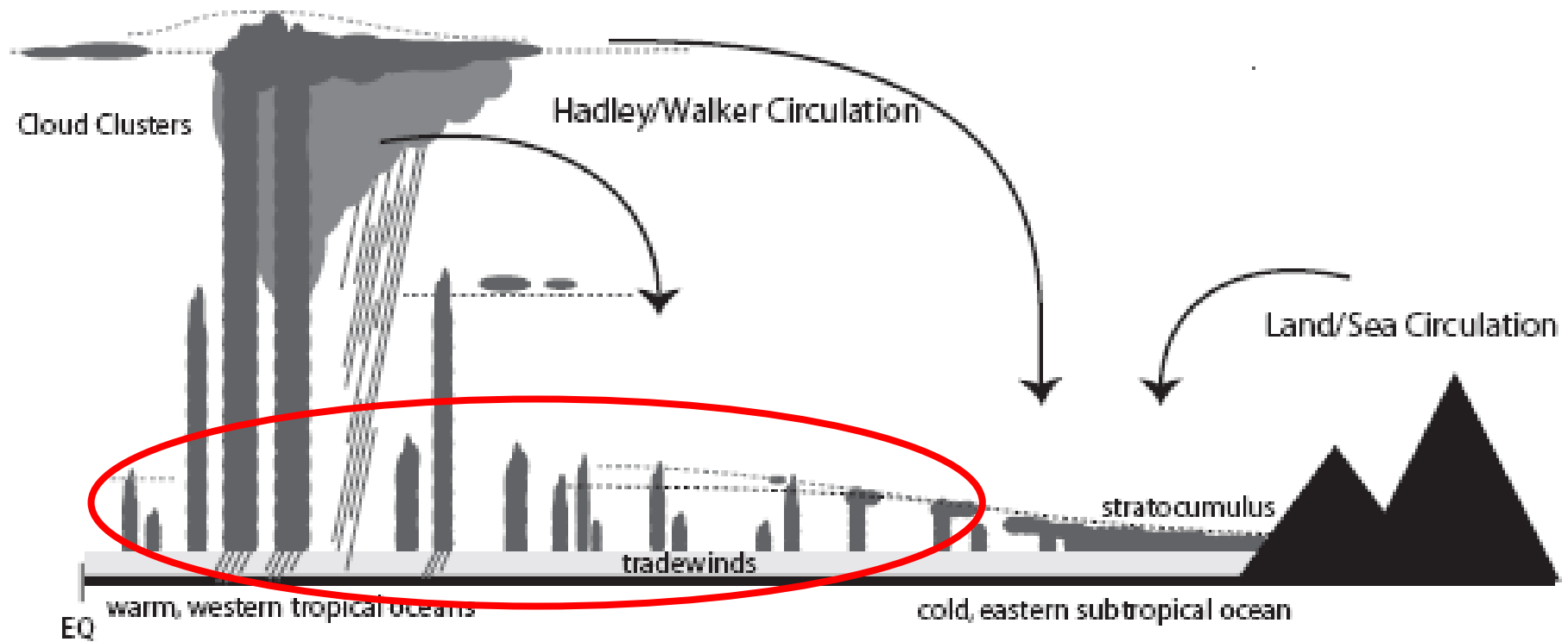
Representation of Cloud Processes in Large Scale Models

Lecture 2

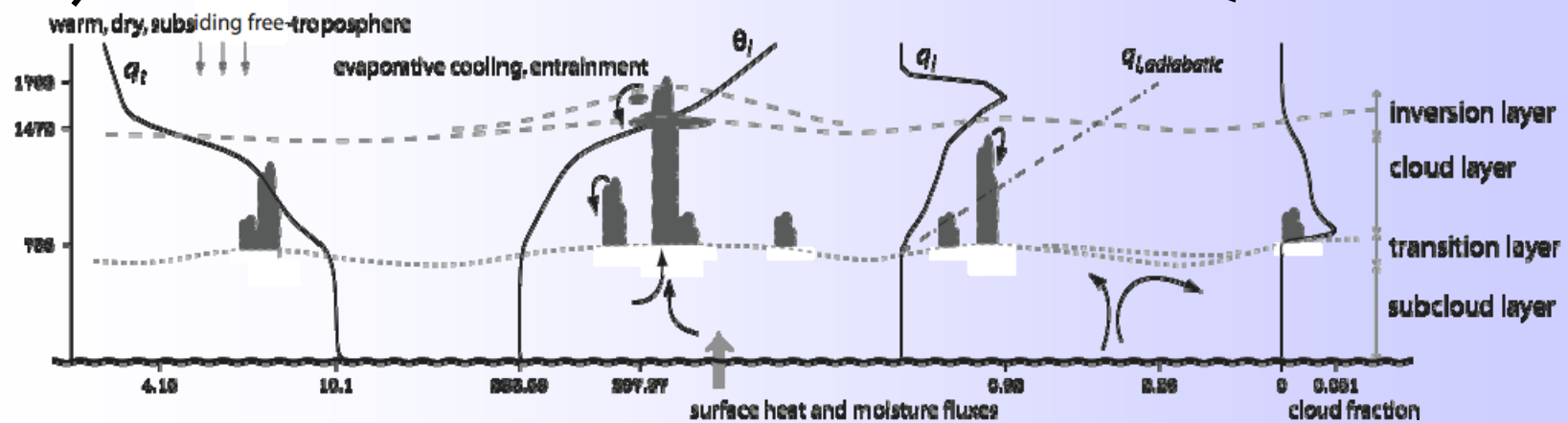
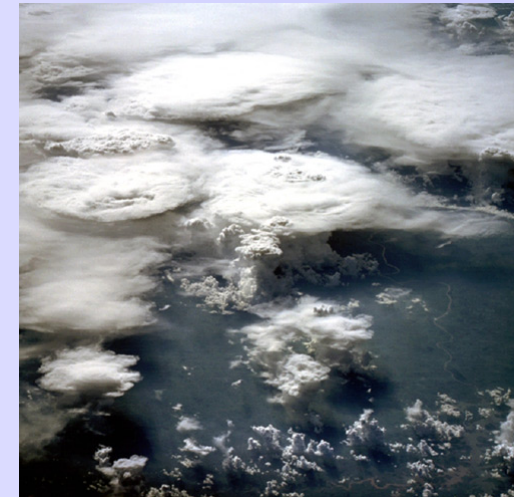
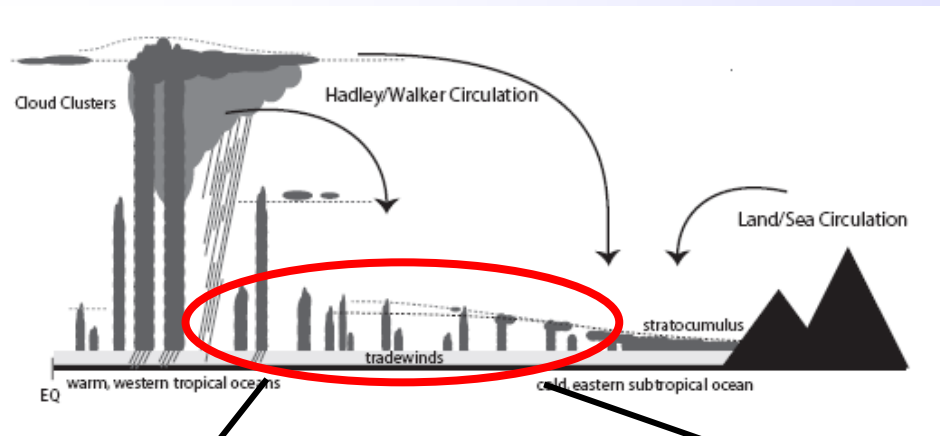
Cumulus Parameterizations

A. Pier Siebesma

The Place of Cumulus Convection



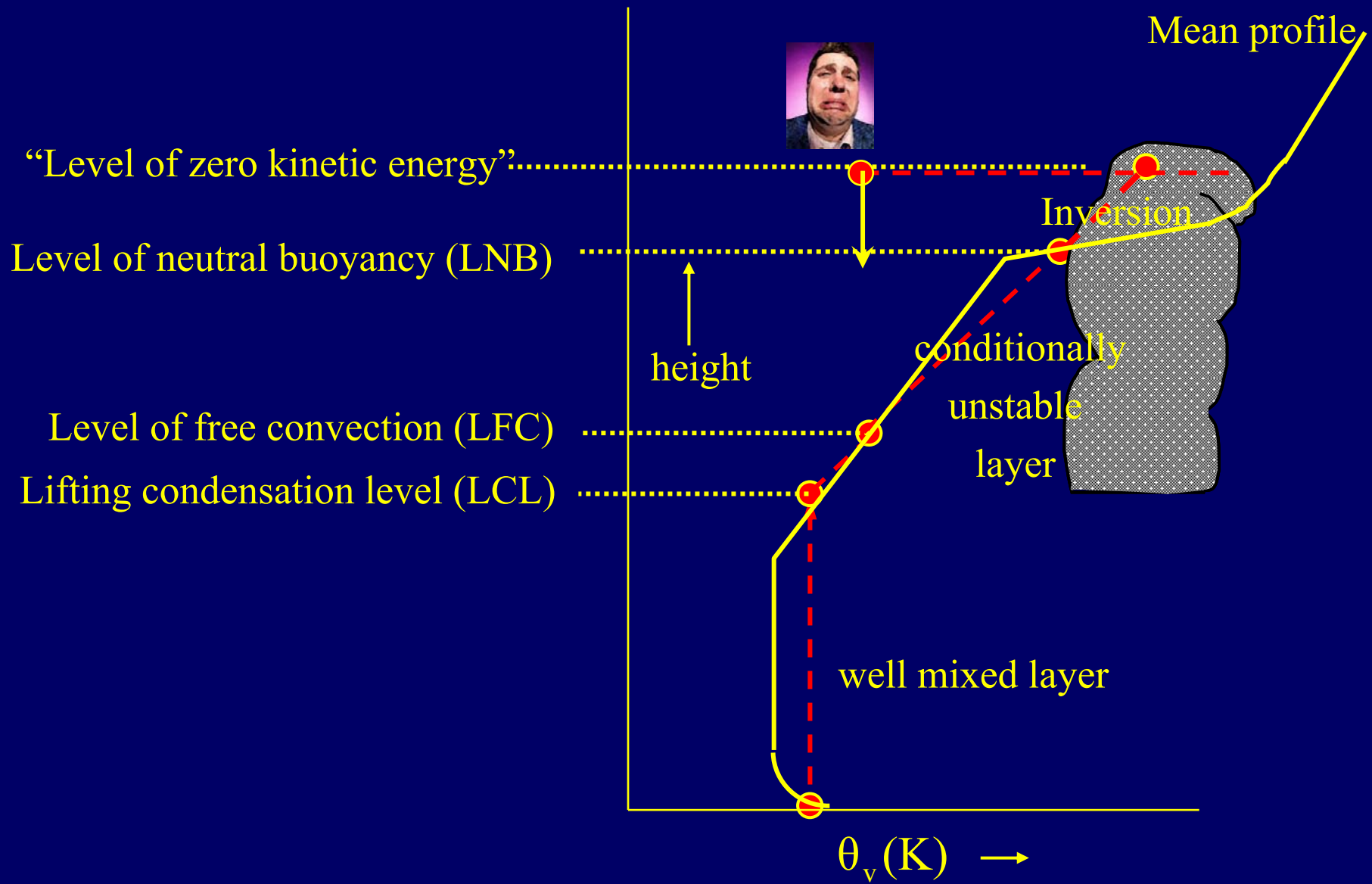
Cumulus Convection



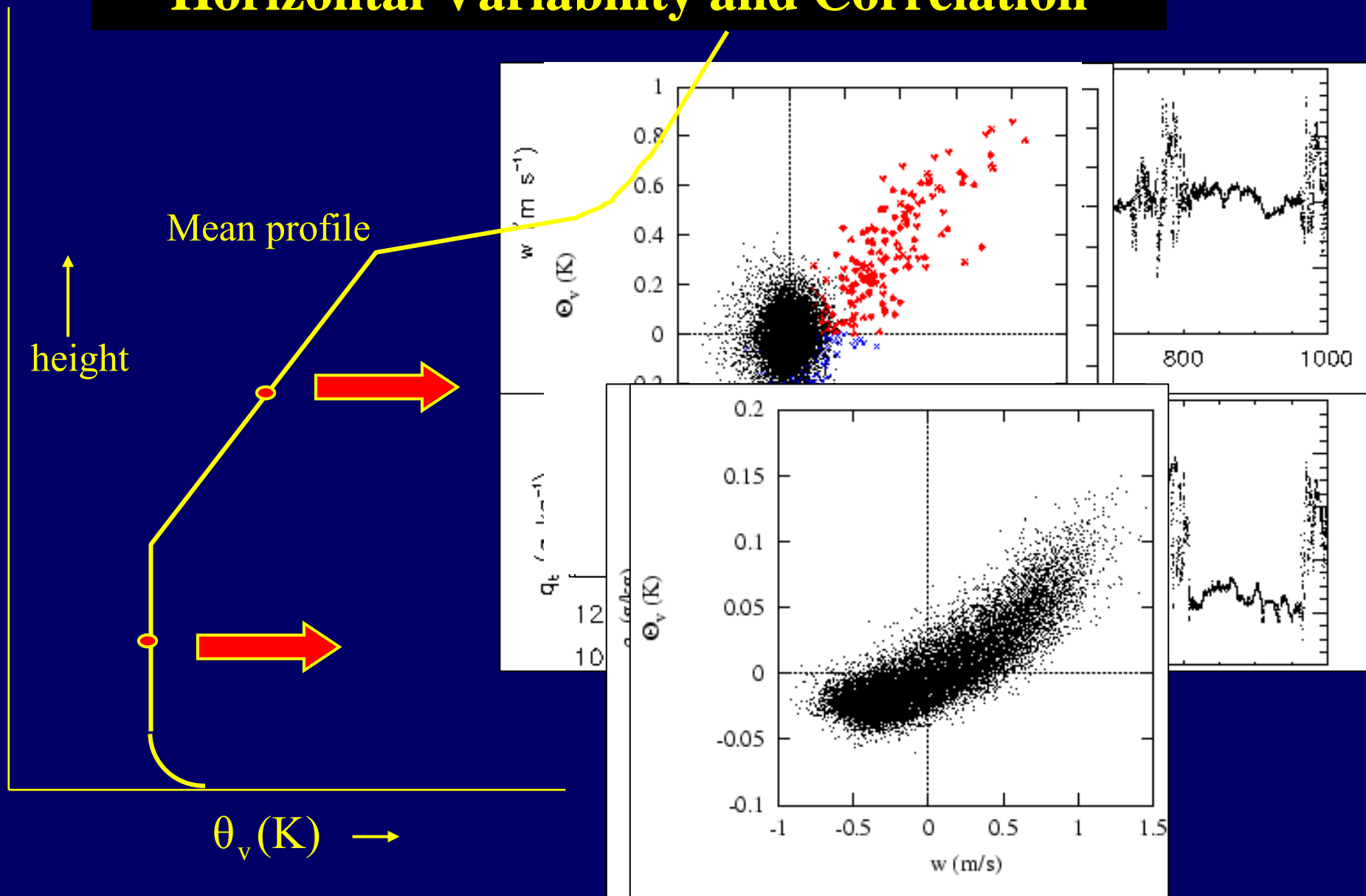
1.

Some unique features of Cumulus Convection

The Miraculous Consequences of conditional Instability

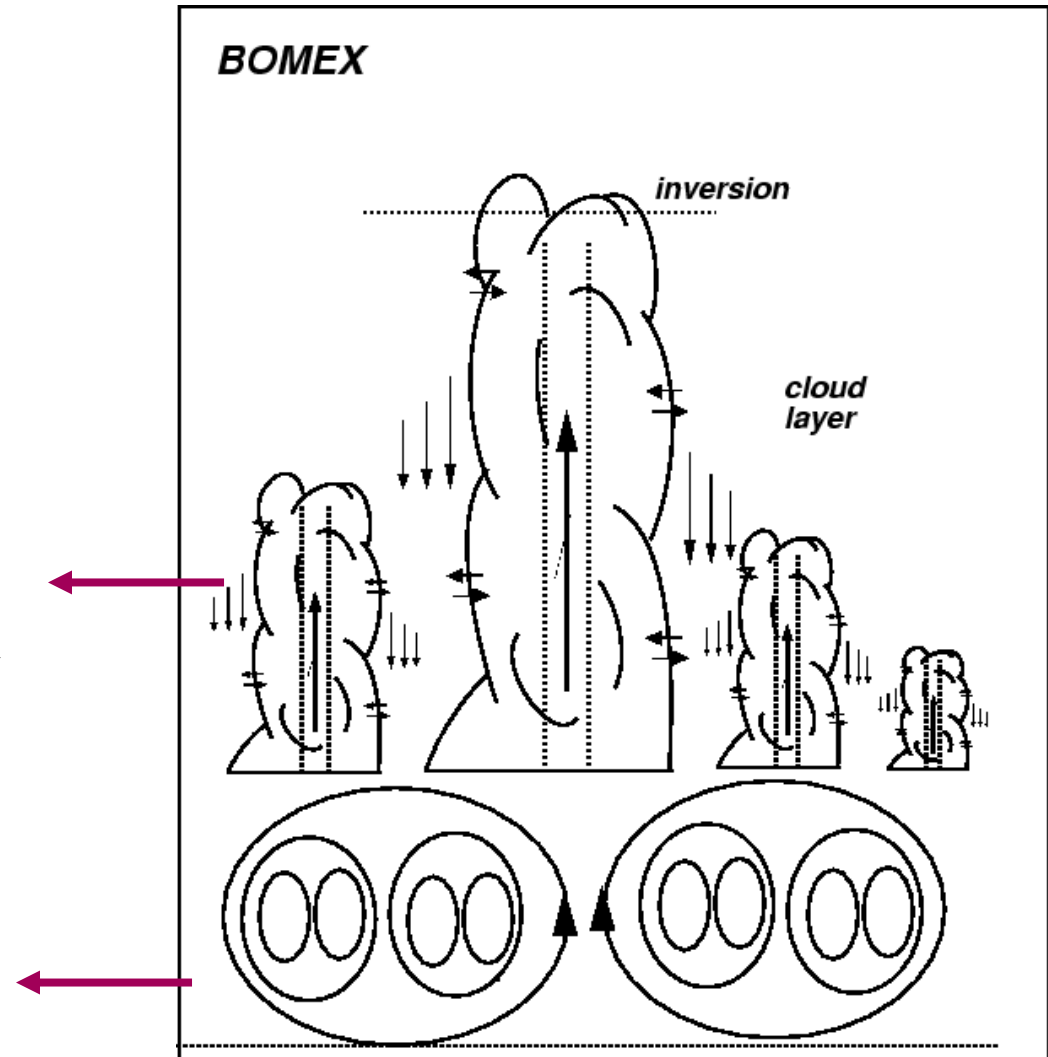


Horizontal Variability and Correlation

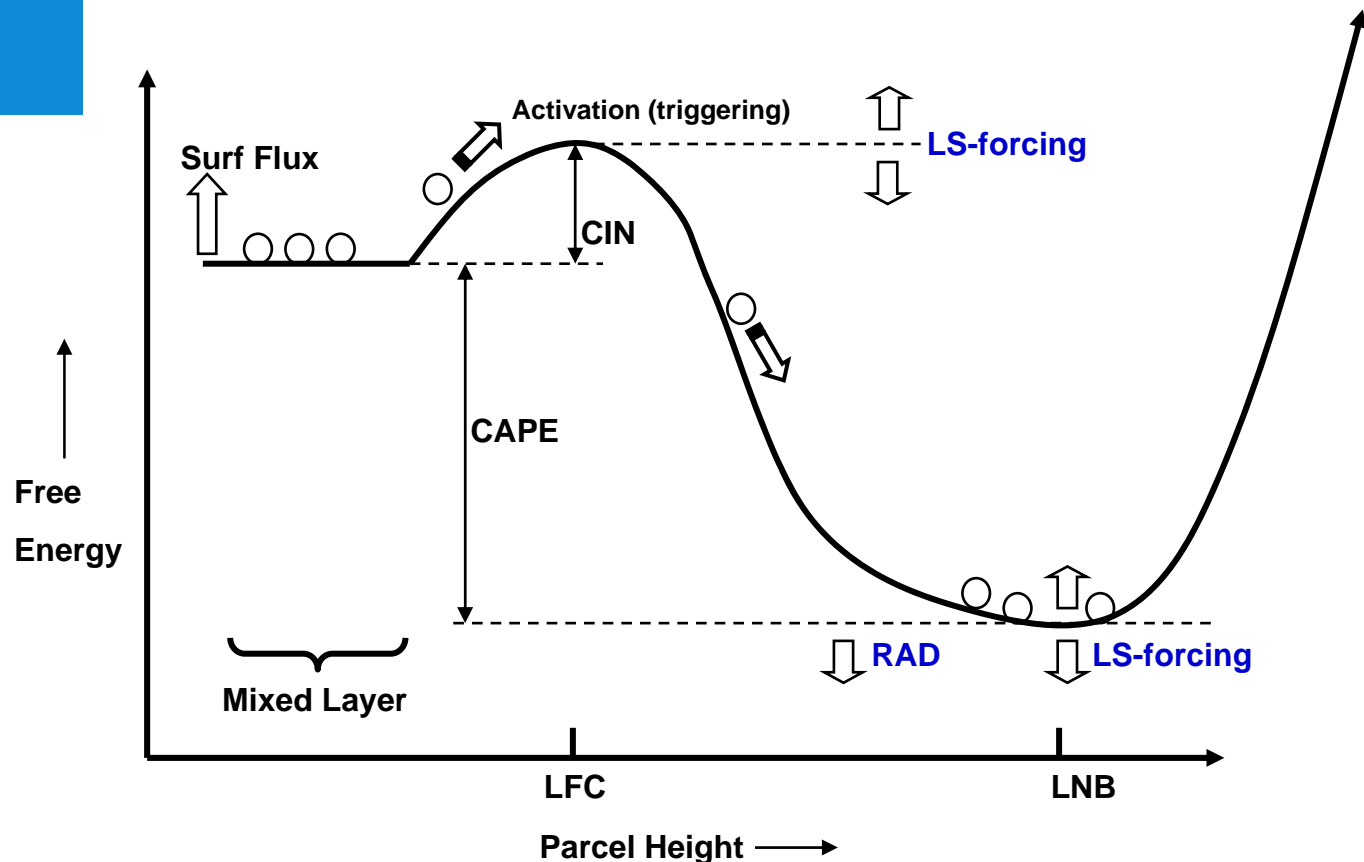


Schematic picture of cumulus moist convection

Cumulus convection:
1. more intermittent
2. more organized
than
Dry Convection.



CAPE and CIN: An Analogue with Chemistry



1) Large Scale Forcing:

- Horizontal Advection
- Vertical Advection (subs)
- Radiation

2) Large Scale Forcing:

slowly builds up CAPE

3) CAPE

- Consumed by moist convection
- Transformed in Kinetic Energy
- Heating due to latent heat release (as measured by the precipitation)
- Fast Process!!

Free after Brian Mapes

Quasi-Equilibrium

$$\frac{dCAPE}{dt} = \frac{CAPE}{\tau_{adj}} + F_{LS} \cong 0$$

LS-Forcing that builds up slowly

The convective process that consumes CAPE
and stabilizes environment

Quasi-equilibrium: near-balance is maintained even when F is
varying with time, i.e. cloud ensemble follows the Forcing.

Forfilled if : $\tau_{adj} \ll \tau_F$

τ_{adj} : hours to a day.



Quasi-equilibrium is (almost) a condition for cumulus convection to be parameterizable

2.

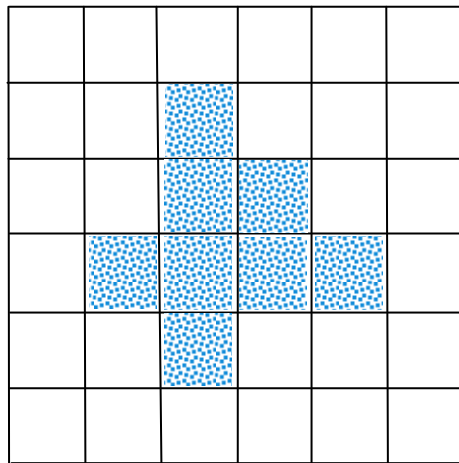
Mass Flux Concept

Conditional Sampling

Introduce an indicator function $c_i = \begin{cases} 1 & \text{if } q_l > 0 \\ 0 & \text{if } q_l = 0 \end{cases}$

Cloud fraction: $\sigma \equiv \frac{1}{N} \sum_{i=1}^N c_i = \frac{N_c}{N}$

average value variable: $\bar{\phi} \equiv \frac{1}{N} \sum_{i=1}^N \phi_i$ with $\phi \in \{\theta_l, q_t, \dots\}$



 cloudy

 clear

in-cloud variable: $\phi_c \equiv \frac{\frac{1}{N} \sum_{i=1}^N c_i \phi_i}{\frac{1}{N} \sum_{i=1}^N c_i} = \frac{1}{N_c} \sum_{i=1}^N c_i \phi_i$

$$\bar{\phi} = \sigma \phi_c + (1 - \sigma) \phi_e$$

For simplicity we assume: $\bar{w} = \sigma w_c + (1 - \sigma) w_e \approx 0$

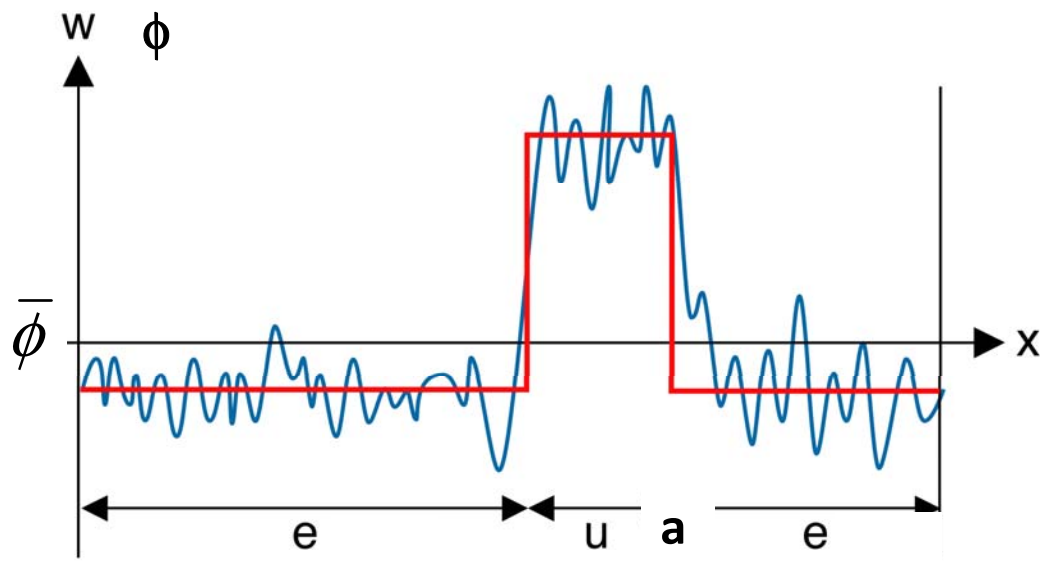
Top-Hat Approximation

$$\overline{w'\phi'} = \frac{1}{N} \sum_{i=1}^N (w_i - \bar{w})(\phi_i - \bar{\phi}) = \frac{1}{N} \sum_{i=1}^N w_i \phi_i - \bar{w} \bar{\phi}$$

Top-hat approximation $\phi_i = \begin{cases} \phi_c & \text{if } q_l > 0 \\ \phi_e & \text{if } q_l = 0 \end{cases}$

$$\overline{w'\phi'} = \sigma w_c (\phi_c - \phi_e)$$

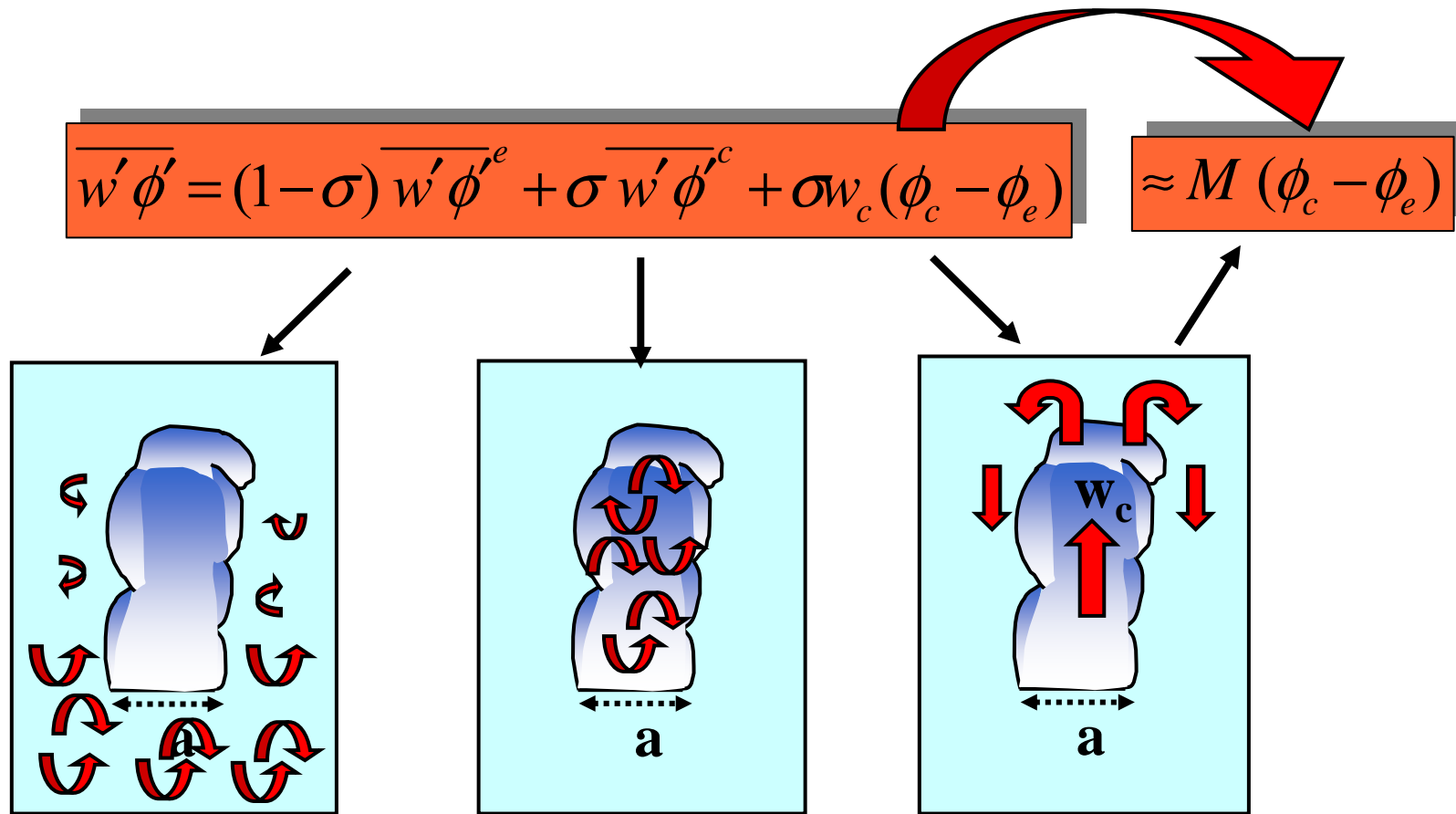
Excercise: derive!



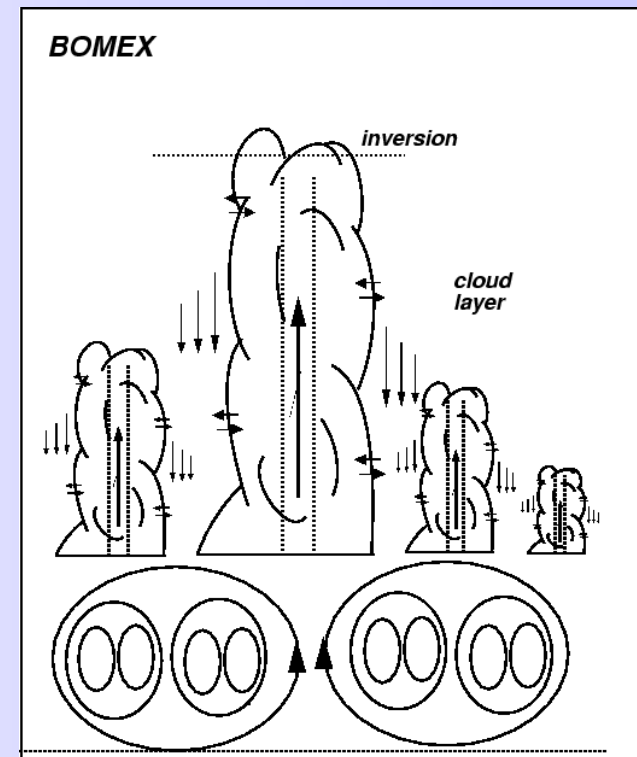
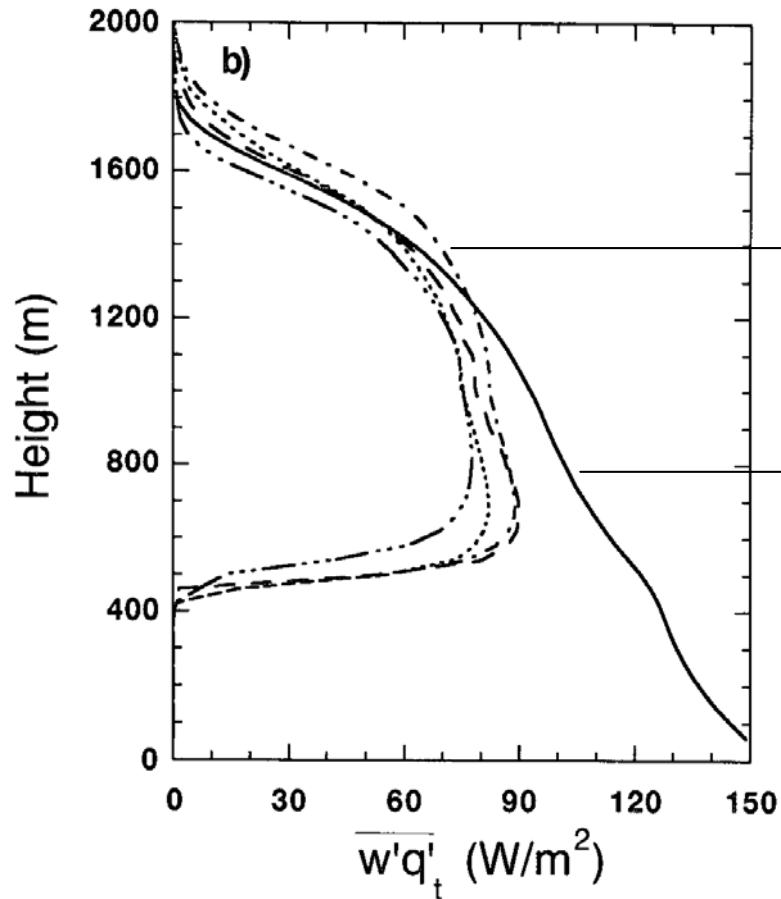
Mass Flux: $M \equiv \sigma w_c$

Strong bimodal character of joint pdf has inspired the design of mass flux parameterizations of turbulent flux in Large scale models

(Betts 1973, Arakawa& Schubert 1974, Tiedtke 1988)



Mass Flux Approximation well validated with LES




3.


Budget Equations using the Mass Flux Approximation

Budget Equations


$$\begin{aligned}
 \frac{\partial \bar{s}_d}{\partial t} &= -\bar{\mathbf{v}} \cdot \nabla \bar{s}_d - \bar{w} \frac{\partial \bar{s}_d}{\partial z} \dots\dots\dots - \frac{\partial}{\partial z} \overline{w' s'_d} + L(c - e) + Q_{rad} \\
 \frac{\partial \bar{q}_v}{\partial t} &= -\bar{\mathbf{v}} \cdot \nabla \bar{q}_v - \bar{w} \frac{\partial \bar{q}_v}{\partial z} \dots\dots\dots - \frac{\partial}{\partial z} \overline{w' q'_v} \dots\dots\dots - (c - e) \\
 \frac{\partial \bar{q}_l}{\partial t} &= -\bar{\mathbf{v}} \cdot \nabla \bar{q}_l - \bar{w} \frac{\partial \bar{q}_l}{\partial z} \dots\dots\dots - \frac{\partial}{\partial z} \overline{w' q'_l} \dots\dots\dots + (c - e) - G
 \end{aligned}$$




Large scale
advection



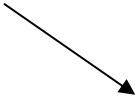
Large scale
subsidence



Vertical
turbulent
transport



Net
Condensation
Rate

Autoconversion to precip 

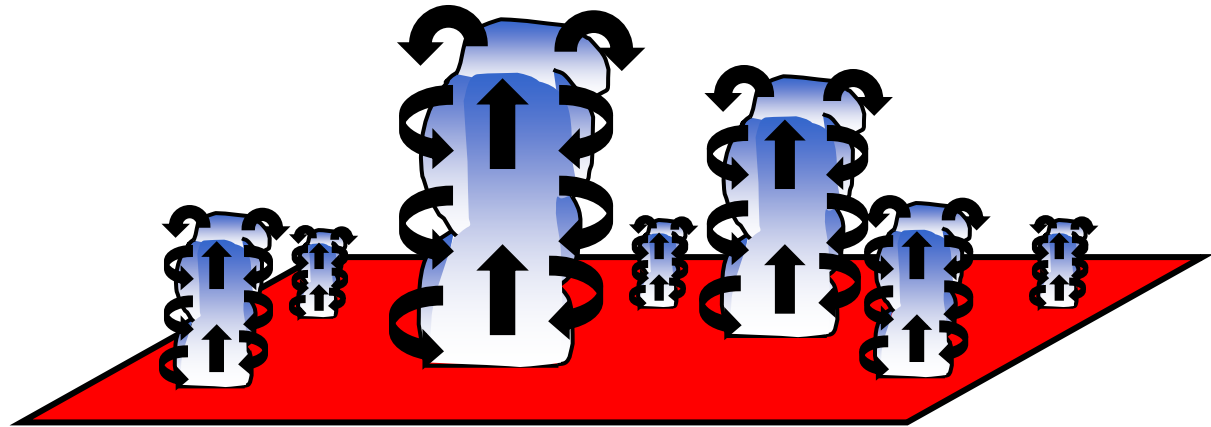
Grid averaged equations for moist conserved variables:

$$\begin{aligned} \frac{\partial \bar{h}_m}{\partial t} &= -\bar{\mathbf{v}} \cdot \nabla \bar{h}_m - \bar{w} \frac{\partial \bar{h}_m}{\partial z} = -\frac{\partial}{\partial z} \overline{w' h'_m} + Q_{rad} \\ \frac{\partial \bar{q}_t}{\partial t} &= -\bar{\mathbf{v}} \cdot \nabla \bar{q}_t - \bar{w} \frac{\partial \bar{q}_t}{\partial z} = -\frac{\partial}{\partial z} \overline{w' q'_t} - G \end{aligned}$$



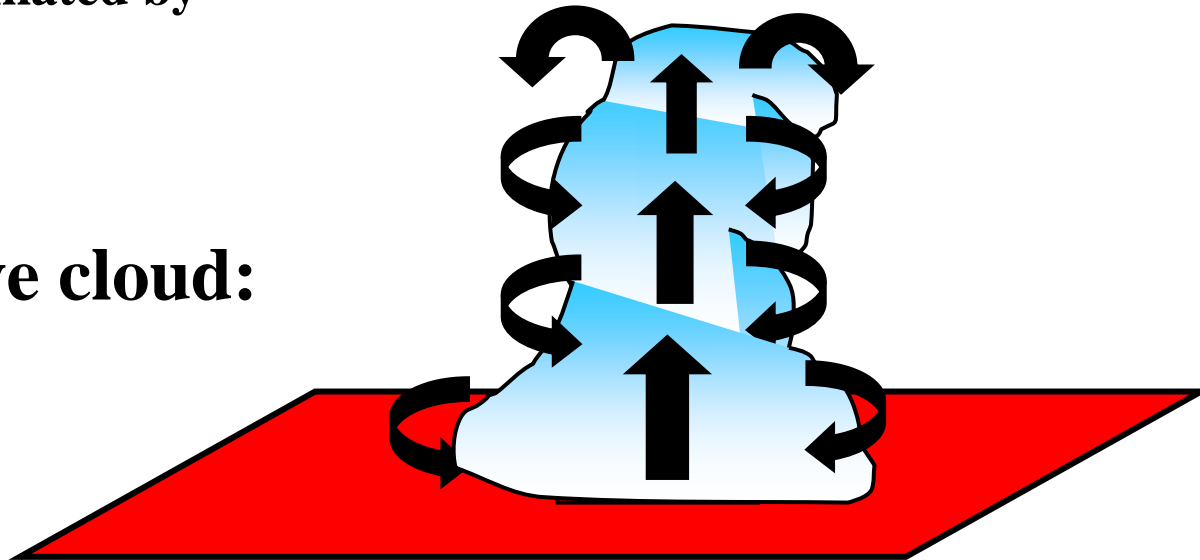
Clouds: use a bulk approach:

Cloud ensemble:



approximated by

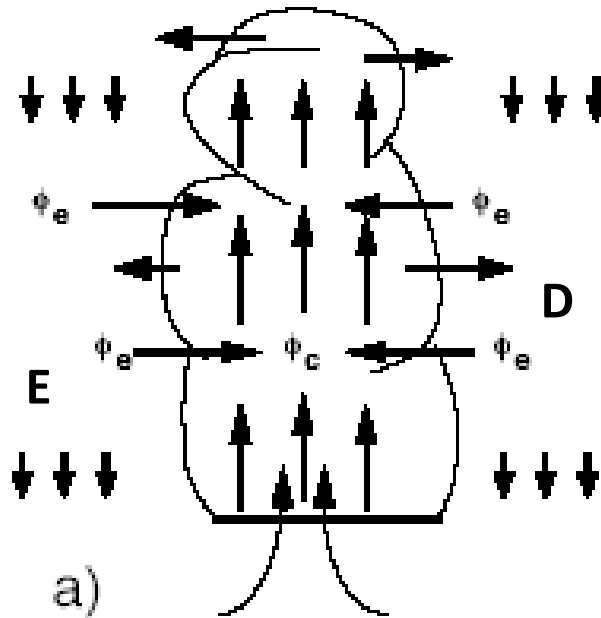
1 effective cloud:



.... Splitting it up in a cloudy and a environmental part

$$\frac{\partial \bar{\phi}}{\partial t} = -\frac{\partial \overline{w'\phi'}}{\partial z} + \bar{F} \approx -\frac{\partial M(\phi_c - \phi_e)}{\partial z} + \bar{F}$$

Separate equations for the cloudy and the clear part:



E: entrainment

D: Detrainment



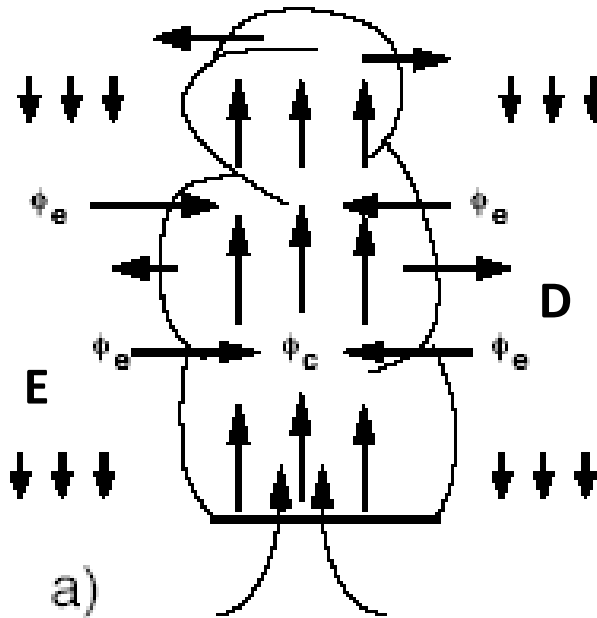
.... Splitting it up in a cloudy and a environmental part

$$\frac{\partial \bar{\phi}}{\partial t} = -\frac{\partial \overline{w'\phi'}}{\partial z} + \bar{F} \approx -\frac{\partial M(\phi_c - \phi_e)}{\partial z} + \bar{F}$$

Separate equations for the cloudy and the clear part:

$$\frac{\partial \sigma \phi_c}{\partial t} = -\frac{\partial M \phi_c}{\partial z} + E \phi_e - D \phi_c + \sigma F_c \quad \text{cloud}$$

$$\frac{\partial (1-\sigma) \phi_c}{\partial t} = \frac{\partial M \phi_e}{\partial z} - E \phi_e + D \phi_c + (1-\sigma) F_e \quad \text{environment}$$



E: entrainment

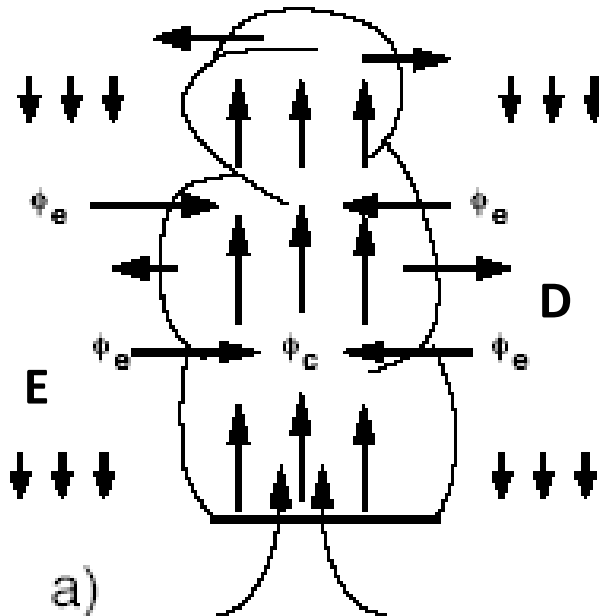
D: Detrainment



Splitting it up in a cloudy and a environmental part

$$\frac{\partial \bar{\phi}}{\partial t} = -\frac{\partial \overline{w'\phi'}}{\partial z} + \bar{F} \approx -\frac{\partial M(\phi_c - \phi_e)}{\partial z} + \bar{F}$$

Separate equations for the cloudy and the clear part:



$$\frac{\partial \sigma \phi_c}{\partial t} = -\frac{\partial M \phi_c}{\partial z} + E \phi_e - D \phi_c + \sigma F_c \quad \text{cloud}$$

$$\frac{\partial (1-\sigma) \phi_c}{\partial t} = \frac{\partial M \phi_e}{\partial z} - E \phi_e + D \phi_c + (1-\sigma) F_e \quad \text{environment}$$

For $\phi=1$ equations reduce to continuity equation:

$$\frac{\partial \sigma}{\partial t} = -\frac{\partial M}{\partial z} + E - D \quad M \equiv a w_c$$

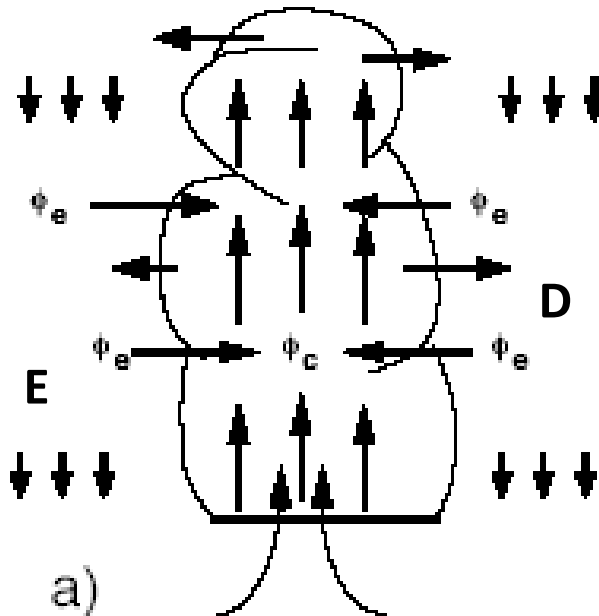
E: entrainment

D: Detrainment

Splitting it up in a cloudy and a environmental part

$$\frac{\partial \bar{\phi}}{\partial t} = -\frac{\partial \overline{w'\phi'}}{\partial z} + \bar{F} \approx -\frac{\partial M(\phi_c - \phi_e)}{\partial z} + \bar{F}$$

Separate equations for the cloudy and the clear part:



$$\frac{\partial \sigma \phi_c}{\partial t} = -\frac{\partial M \phi_c}{\partial z} + E \phi_e - D \phi_c + \sigma F_c \quad \text{cloud}$$

$$\frac{\partial (1-\sigma) \phi_c}{\partial t} = \frac{\partial M \phi_e}{\partial z} - E \phi_e + D \phi_c + (1-\sigma) F_e \quad \text{environment}$$

For $\phi=1$ equations reduce to continuity equation:

$$\frac{\partial \sigma}{\partial t} = -\frac{\partial M}{\partial z} + E - D \approx 0 \quad M \equiv a w_c$$

Approximation 1: $\frac{\partial \sigma \phi_c}{\partial t} \approx 0$

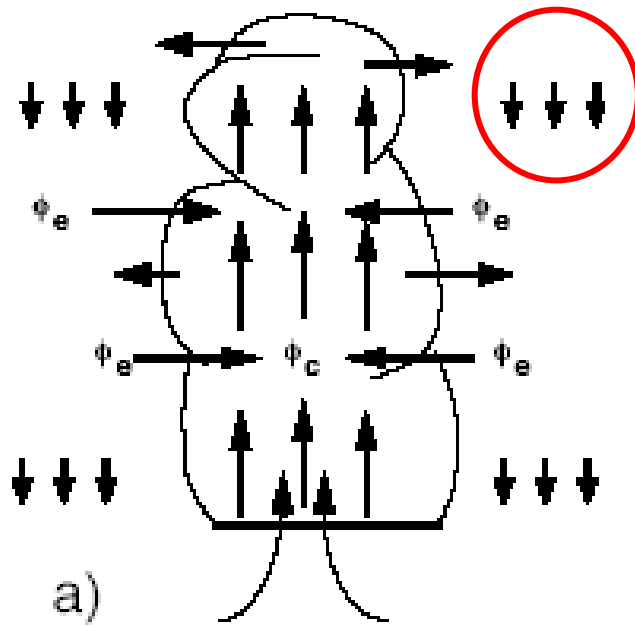
Approximation 2: $\sigma \ll 1$ implying $\phi_e \approx \bar{\phi}$

E: entrainment

D: Detrainment

How does the cloud ensemble influence the environment?

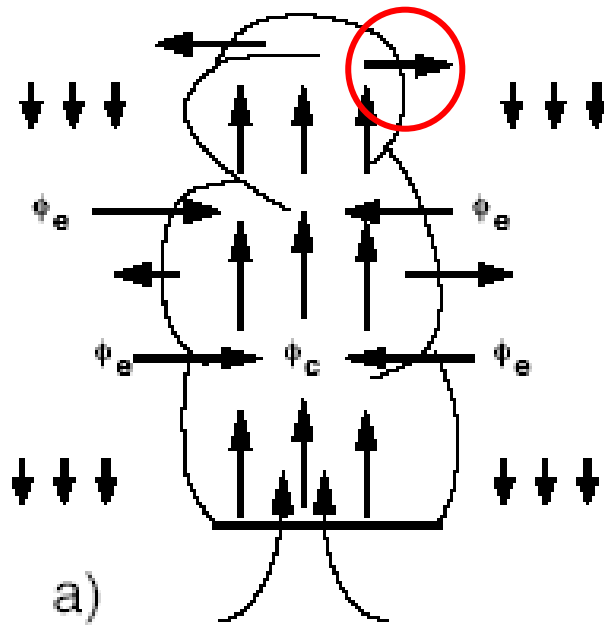
$$\left(\frac{\partial \bar{\phi}}{\partial t} \right)_{conv} = M \frac{\partial \bar{\phi}}{\partial z} + D \phi_c$$



Compensating Subsidence (warming and drying)

How does the cloud ensemble influence the environment?

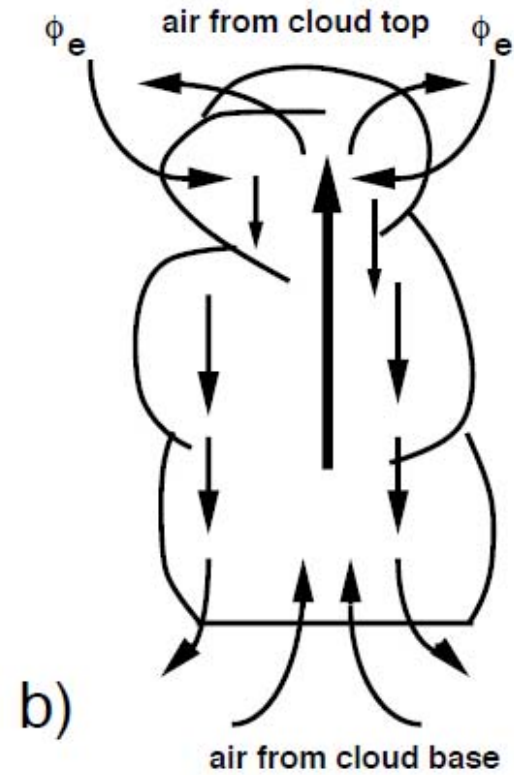
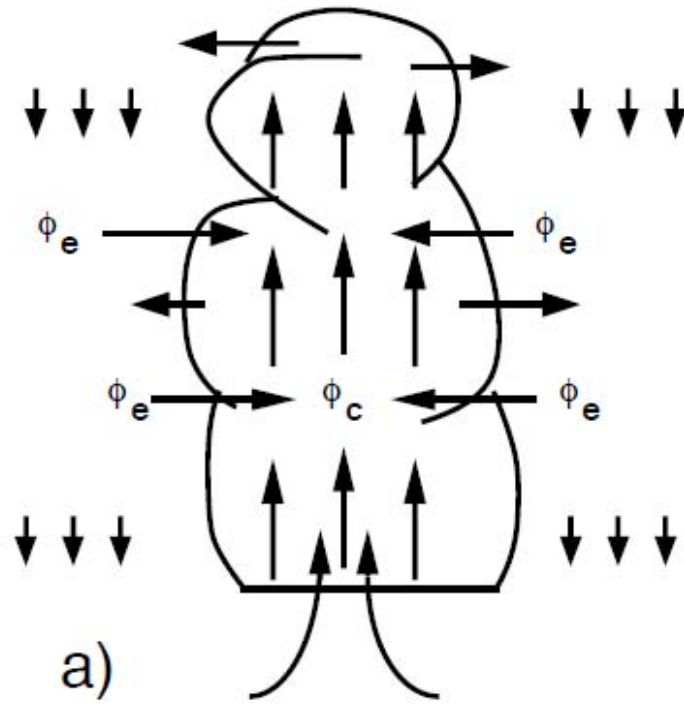
$$\left(\frac{\partial \bar{\phi}}{\partial t}\right)_{conv} = M \frac{\partial \bar{\phi}}{\partial z} + D\phi_c$$



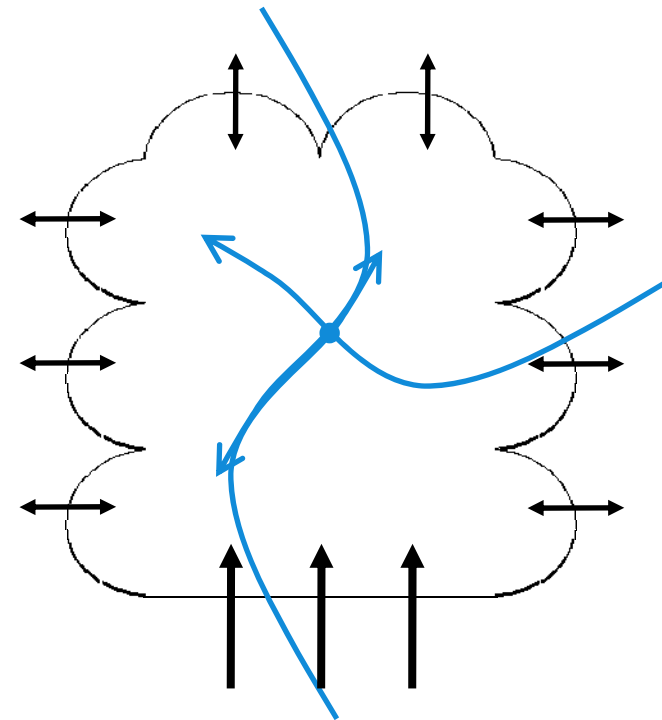
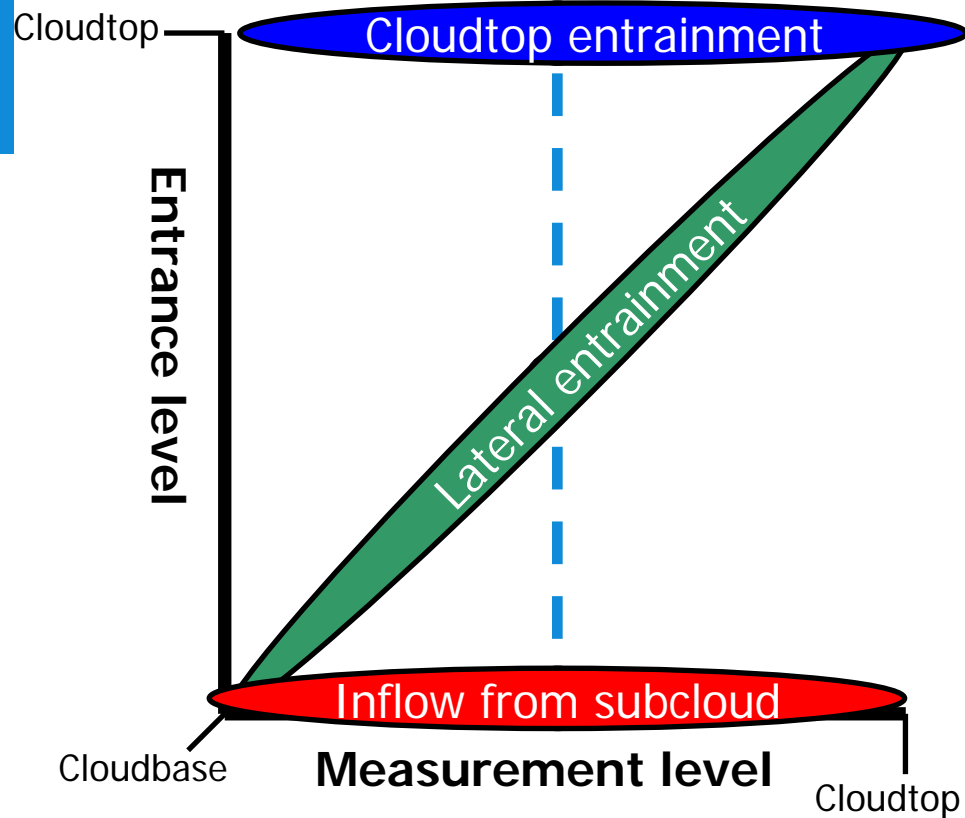
Detrainment (Cooling and Moistening)

4.

Vertical versus Lateral Mixing



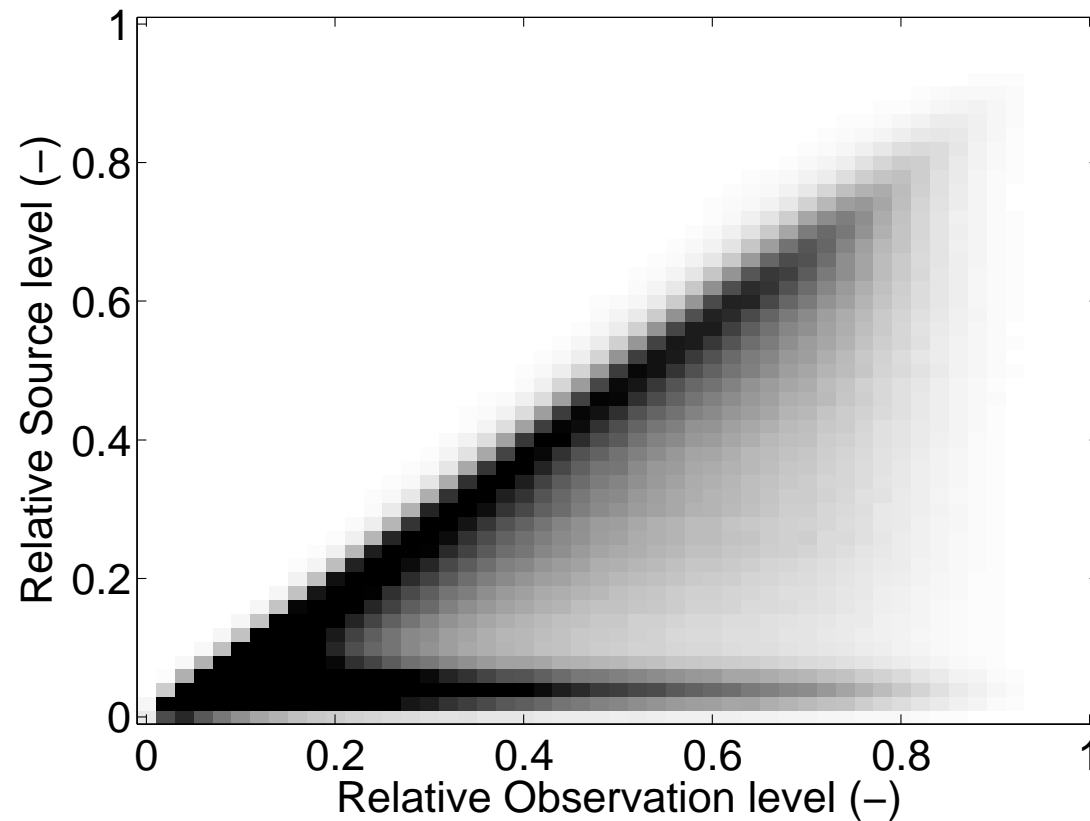
Backtracing particles in LES: where does the air in the cloud come from?



(Heus et al, 2008 JAS)

Height vs. Source level

Core Parcels, Cloud Size > 1000 m
Non-Normalized



Virtually all cloudy air comes from below the observational level!!

5.

Mass Flux Scheme in action

How to make an updraft model

Prognostic equation:
$$\frac{\partial \bar{\phi}}{\partial t} \approx -\frac{\partial M(\phi_c - \bar{\phi})}{\partial z} + \bar{F}$$
 with $\bar{\phi} = \{\theta_l, q_t\}$

Continuity equation:
$$\frac{\partial M}{\partial z} = E - D$$

Steady state cloud eq,:
$$\frac{\partial M\phi_c}{\partial z} = -E\bar{\phi} + D\phi_c$$

$$\frac{\partial \phi_c}{\partial z} = -\varepsilon(\phi_c - \bar{\phi})$$

$$\frac{\partial \ln M}{\partial z} = \varepsilon - \delta$$

Introduce fractional rates: ε : fractional entrainment : $\varepsilon=E/M$
 δ : fractional detrainment: $\delta=D/M$

Excercise!!

Implementation simple bulk mass flux scheme

1. Updraft Calculation in conserved variables:

$$\frac{\partial \theta_{l,c}}{\partial z} = -\varepsilon (\theta_{l,c} - \bar{\theta}_l)$$

$$\frac{\partial q_{t,c}}{\partial z} = -\varepsilon (q_{t,c} - \bar{q}_t)$$

2. Reconstruct non-conserved variables:

$$\{\theta_l, q_t\} \Rightarrow \{\theta, \theta_v, q_v, q_l, \}$$

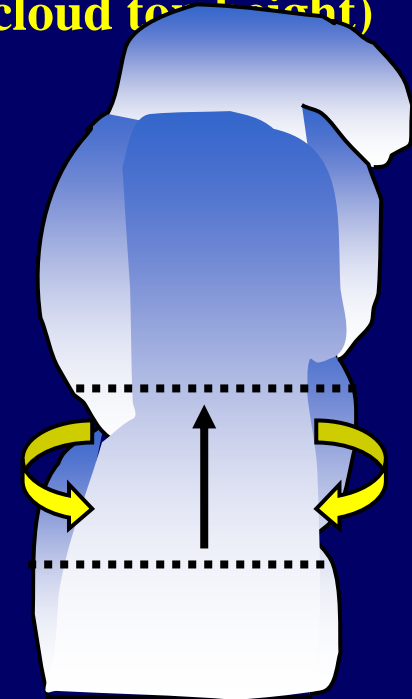
continue

B > 0

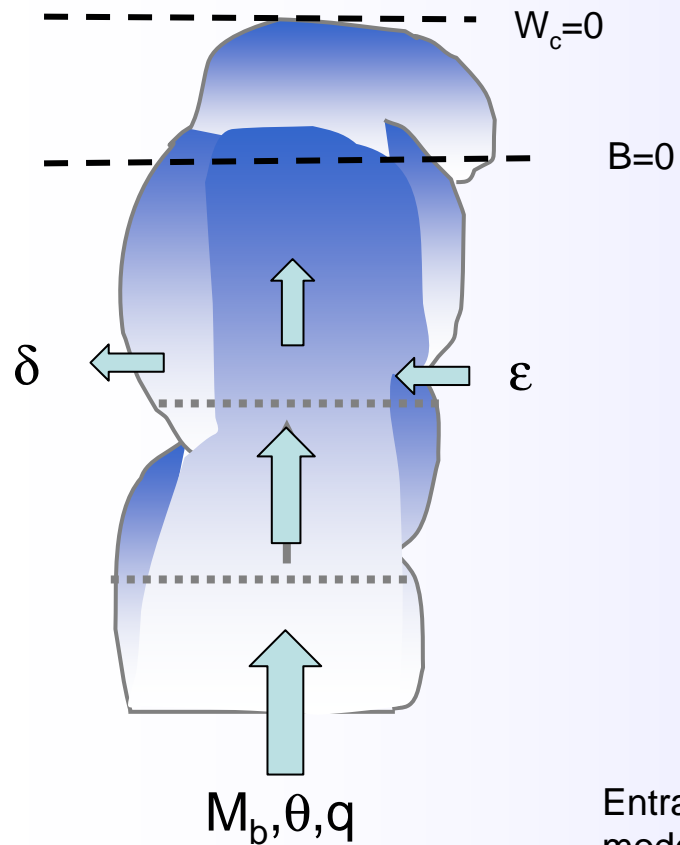
Stop (= cloud top height)

3. Check on Buoyancy:

$$B = \frac{g}{\theta_v} (\theta_{v,c} - \bar{\theta}_v)$$



All nice and fine, but.....



We need to know:

1. What is the entrainment and detrainment
2. At which height does the cloud stop (wc-equation)
3. What are the values of M, θ, q at cloud base (Closure)
4. When does convection initiate (triggering)

Entrainment is one of the most sensitive parameters in climate models.....

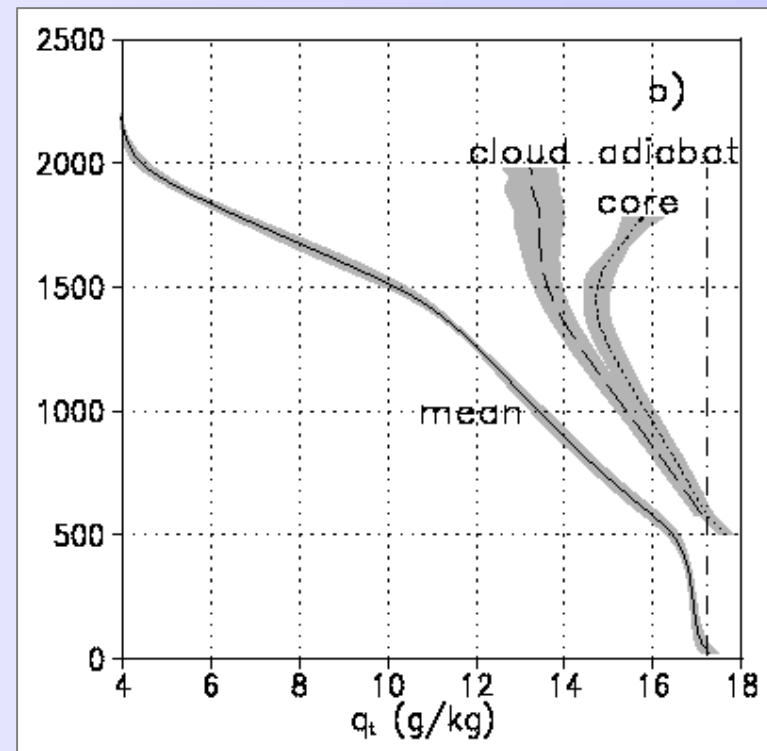
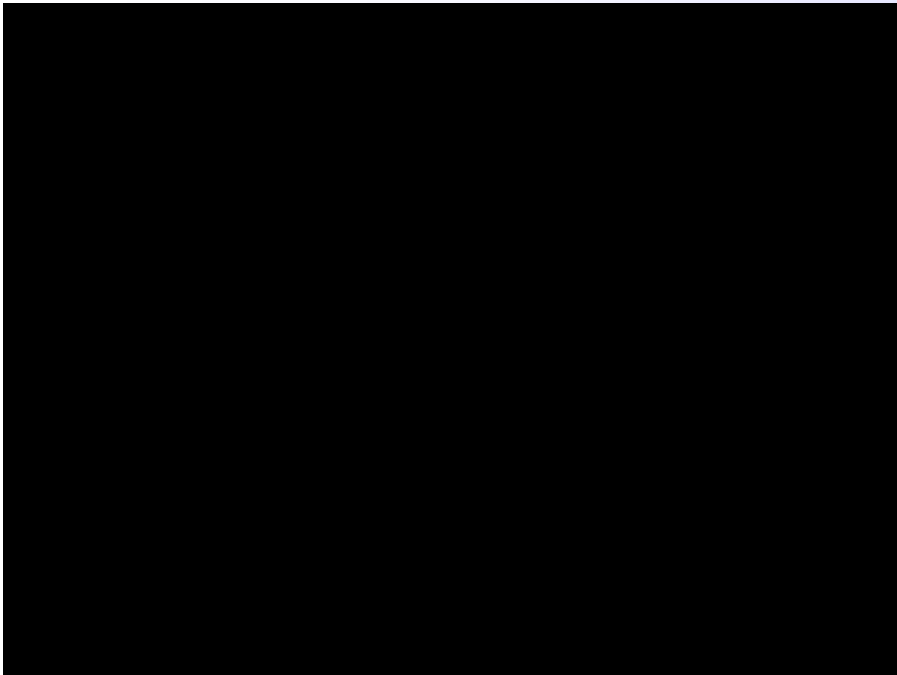
8.

Entrainment and Detrainment

Entrainment

- Read de Rooy et al. Entrainment and detrainment in cumulus convection: an overview, QJRMS (2013)

LES



$$\frac{\partial \phi_c}{\partial z} = -\varepsilon(\phi_c - \bar{\phi})$$

Early Plume models (1)

Continuity Equation

$$\oint v dl + \frac{\partial A_{plume} w_{plume}}{\partial z} = 0$$

Assume circular geometry:

$$2\pi R v_r + \frac{\partial \pi R^2 w_p}{\partial z} = 0$$

$$\frac{2\pi R^2 v_r}{R} + \frac{\partial \pi R^2 w_p}{\partial z} = 0$$

$$M \equiv \pi R^2 w_p \quad \Rightarrow$$

$$\frac{2}{R} \frac{v_r}{w_p} M + \frac{\partial M}{\partial z} = 0$$

Scaling Ansatz : $v_r \cong \alpha w_c$

$$\frac{1}{M} \frac{\partial M}{\partial z} = -\frac{2\alpha}{R} \quad \text{or} \quad \varepsilon = \frac{2\alpha}{R} \quad \text{with} \quad \alpha \approx 0.1$$

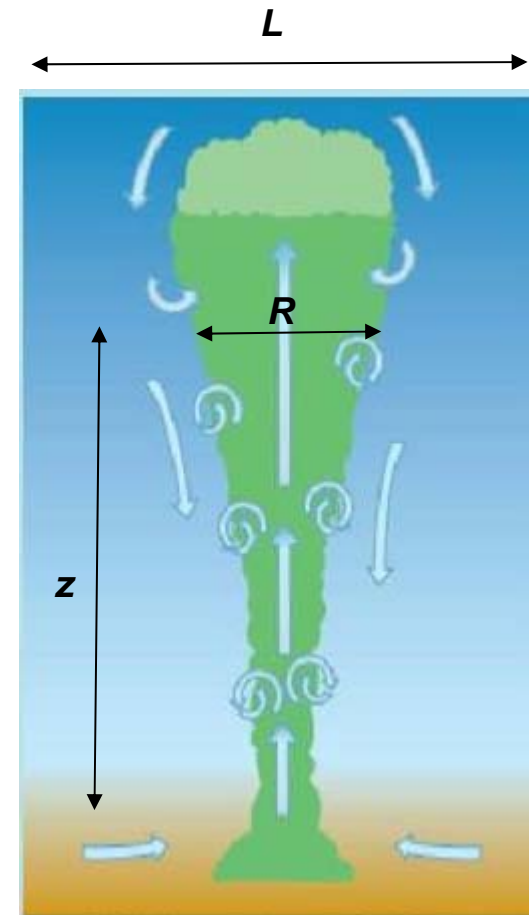


Figure 9-6. The column or plume model of a thermal.

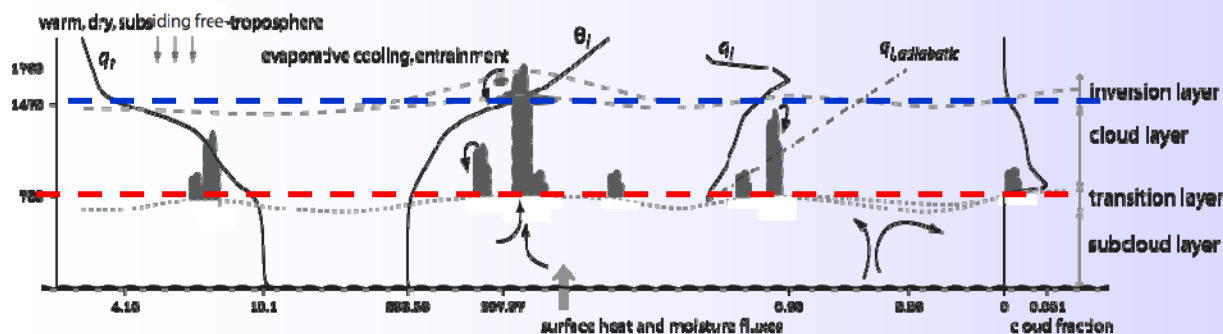
- From plume models: $\varepsilon = \frac{2\alpha}{R}$ Essentially a dimensional argument

- Qualitatively correct : Wider clouds have a smaller entrainment rate.

 - Also: "Deeper clouds have a smaller entrainment rates.

- Typical values: $\varepsilon \approx z^{-1}$ Shallow clouds (1km depth): $\varepsilon \approx 10^{-3} m^{-1}$
Deep clouds (10km depth): $\varepsilon \approx 10^{-4} m^{-1}$

- How to choose typical values for a cloud ensemble?



Dominated by large clouds

Dominated by small clouds

Shallow Cumulus Convection

Siebesma JAS 2003

LES



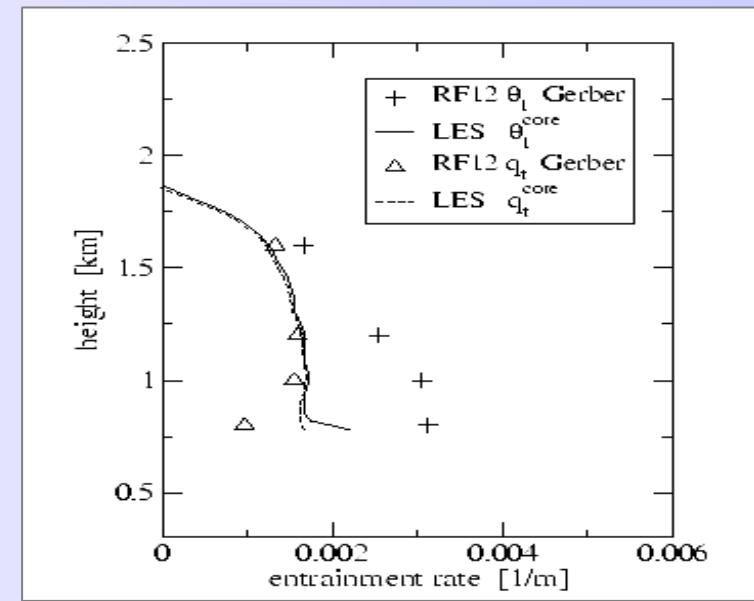
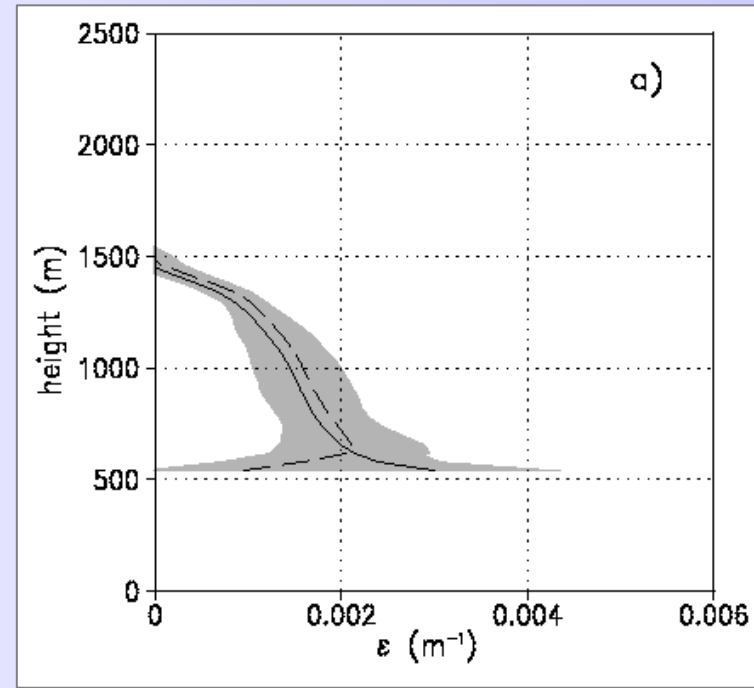
$$\varepsilon = 1 \sim 3 \cdot 10^{-3} \text{ m}^{-1}$$



Observations

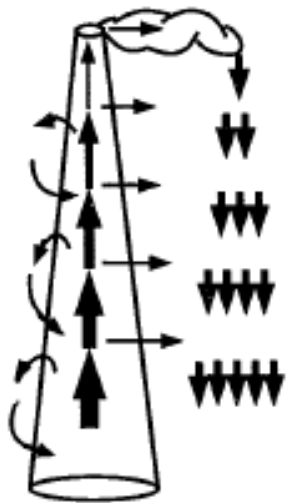
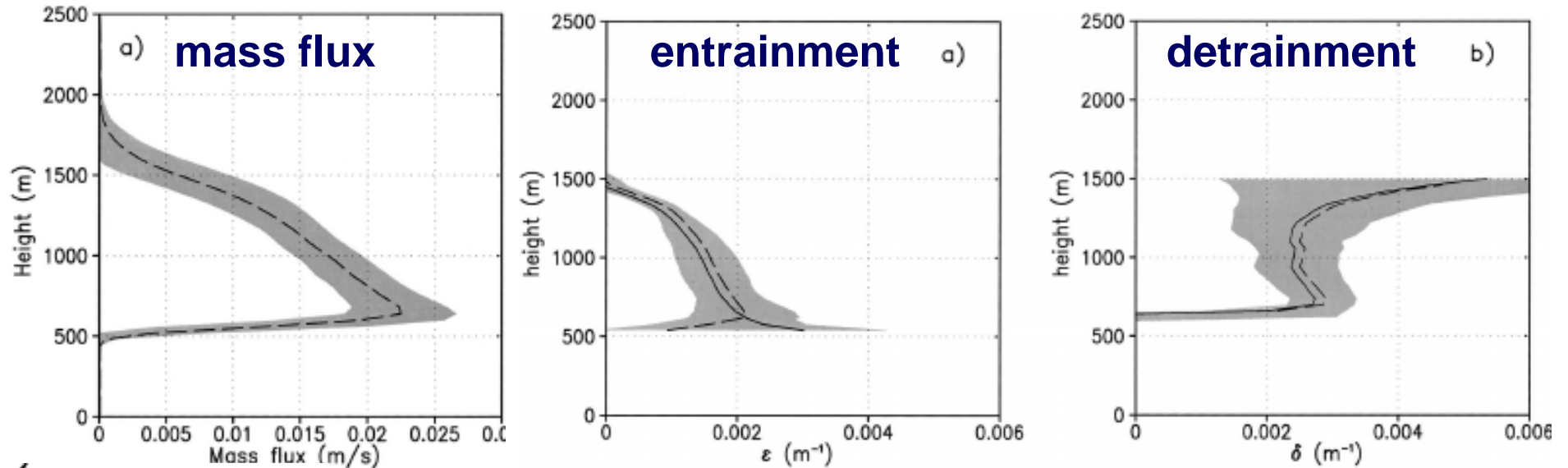


Cumulus over Florida: SCMS



Courtesy Stephan Rodts TU Delft

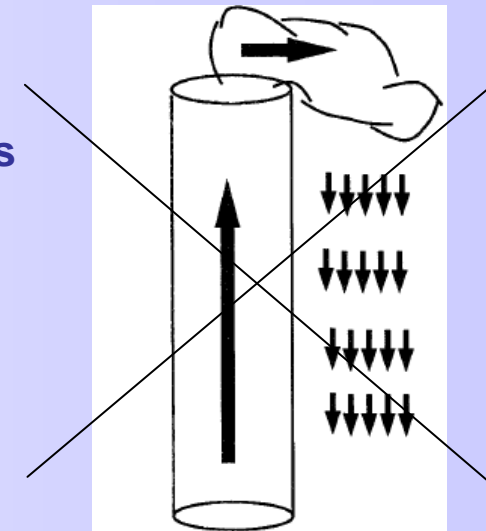
Steady State shallow cumulus (BOMEX). LES results:



Main Results:

1. Lateral entrainment and detrainment rates typically of the order of $10^{-3} m^{-1}$
2. Detrainment rates typically larger than entrainment rates or
3. Mass flux decreases with height

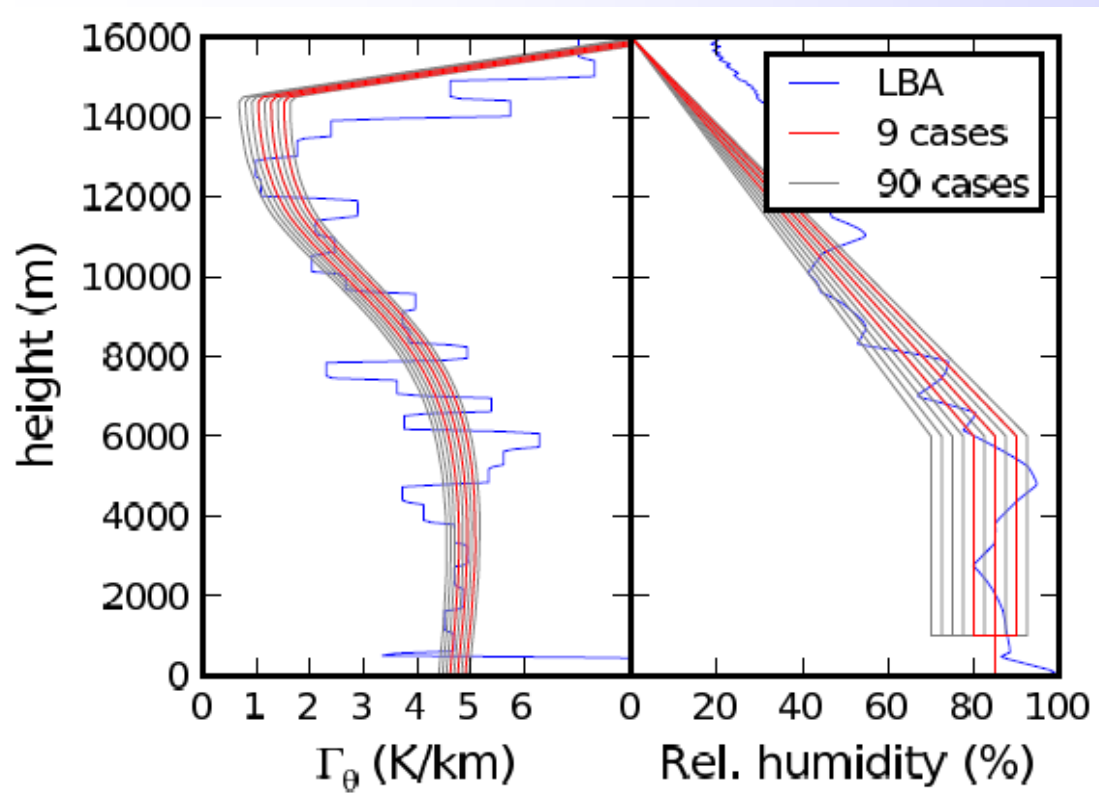
Siebesma et al (1995, 1996, 2003)



How about Deep Convection?

(Boing et al GRL 2013)

Similar set up as in: Wu, Stevens, Arakawa JAS 2009

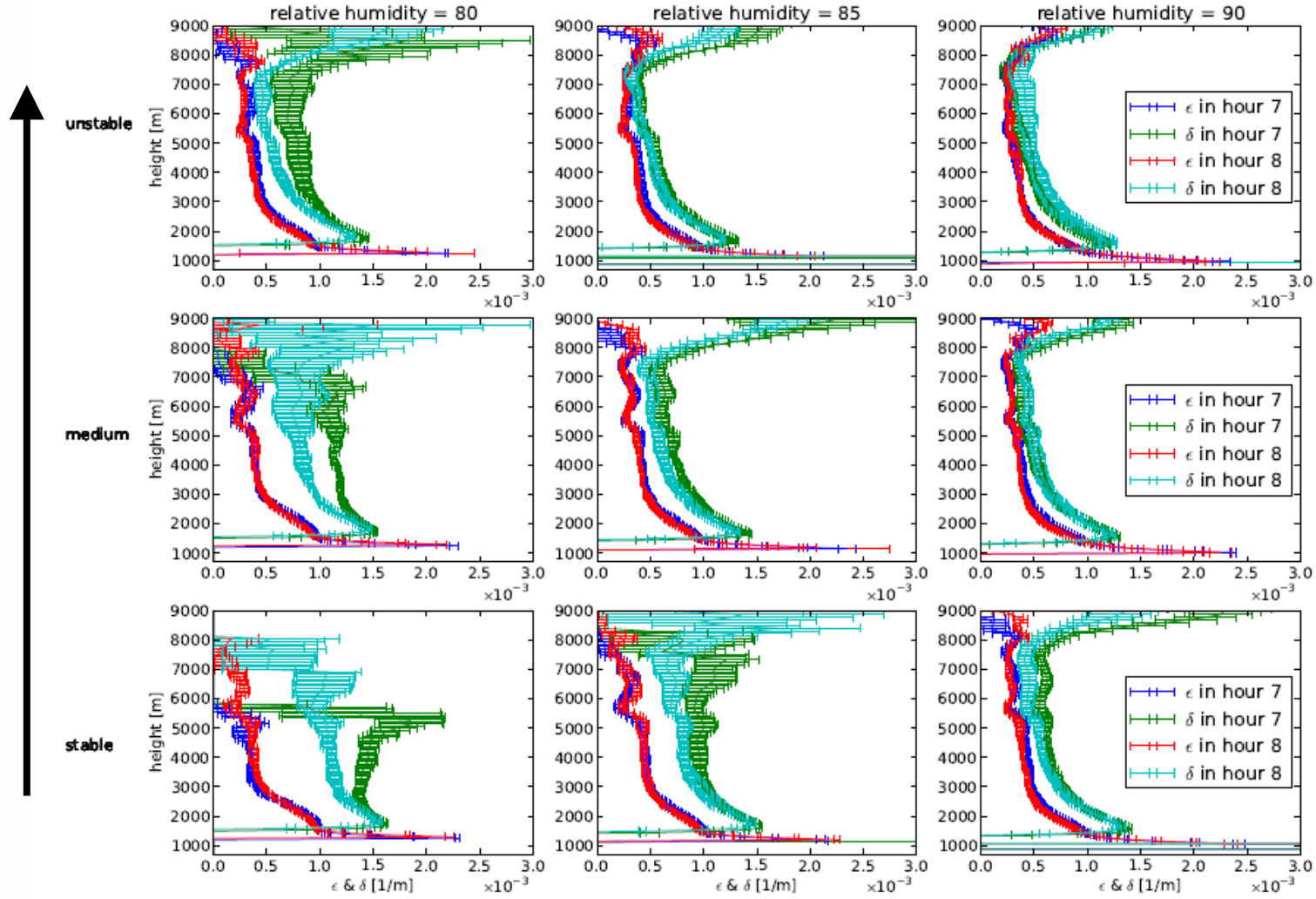


- Domain Size 75X75X25km
- $\Delta x = \Delta y = 150\text{m}$ $\Delta z = 40 \sim 190\text{m}$
- Fixed surface fluxes:
 - LHF $\sim 350\text{W/m}^2$
 - SHF $\sim 150\text{W/m}^2$
- No windshear
- No radiation

Most cases repeated 5 times with different random initialisation (200 simulations)

Red and Blue lines : entrainment rates (LES results)

More unstable



Moister environment

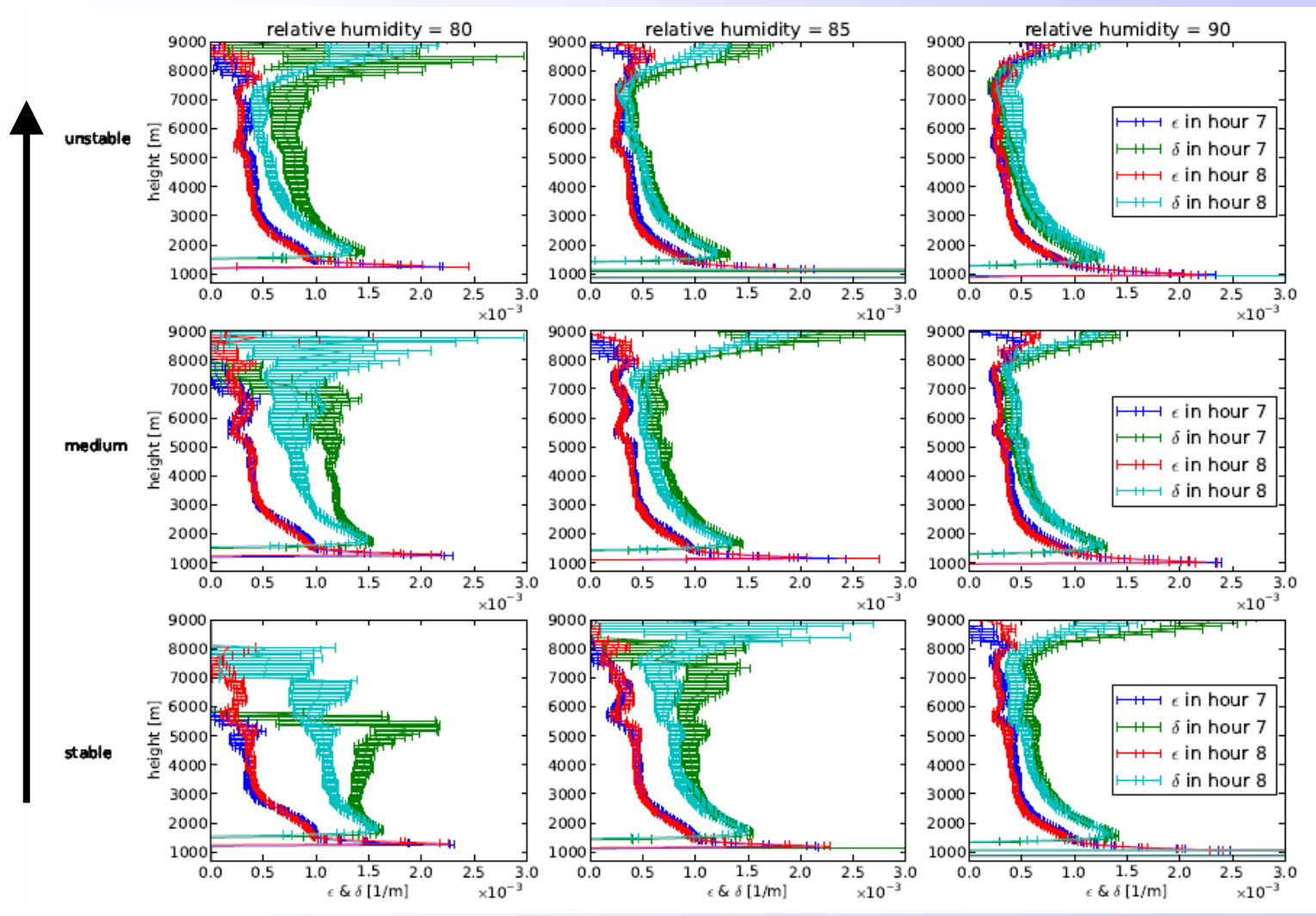
But hey, what about Detrainment??

$$\frac{\partial \ln M}{\partial z} = \varepsilon - \delta$$

$$\frac{\partial \bar{\phi}}{\partial t} \approx -\frac{\partial M(\phi_c - \bar{\phi})}{\partial z} + \bar{F}$$

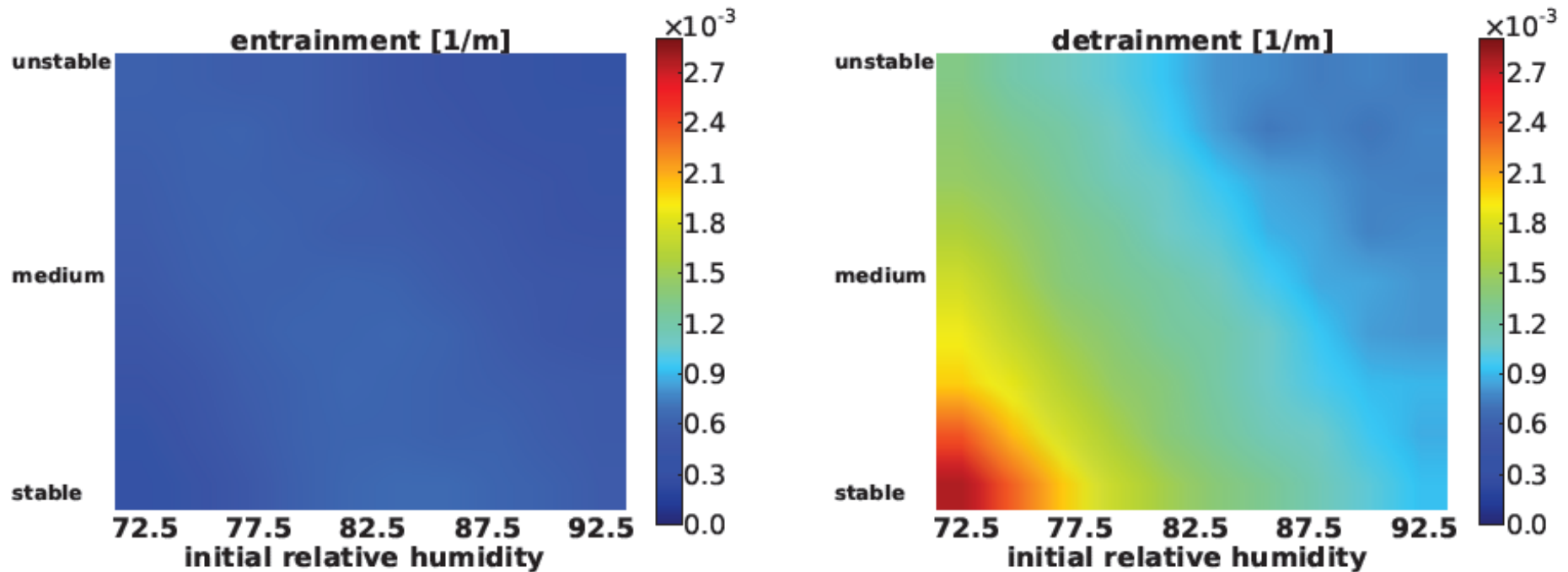
But hey, what about Detrainment??

More unstable



moister

entrainment and detrainment (2000~3000m)



- Detrainment decreases with increasing humidity
- Detrainment decreases with increasing instability
- Variations of Entrainment small.....compared with the variations of detrainment

But hey, what about Detrainment??

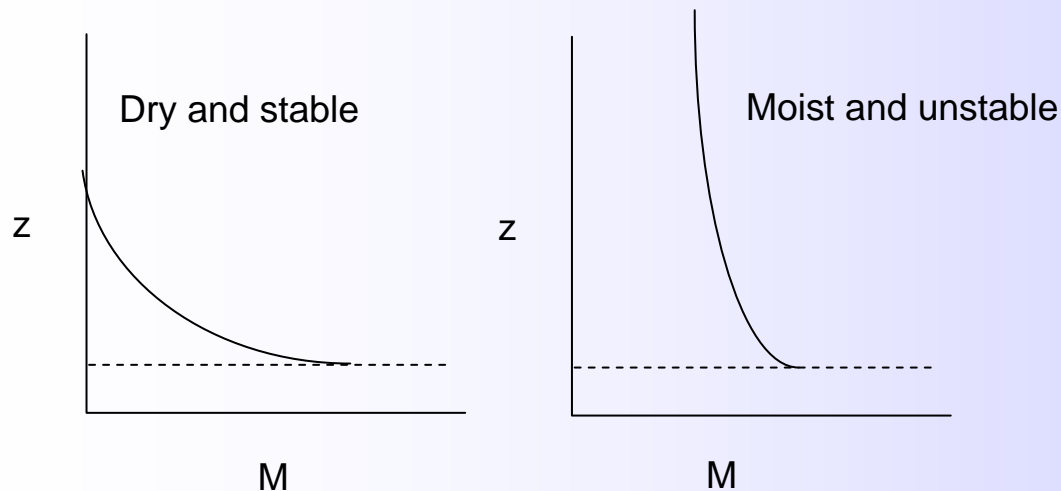
Simulations show that detrainment:

- becomes smaller in a more unstable environment!
- becomes smaller in a more humid environment!

$$\frac{\partial \ln M}{\partial z} = \varepsilon - \delta$$

Towards values comparable to the entrainment.

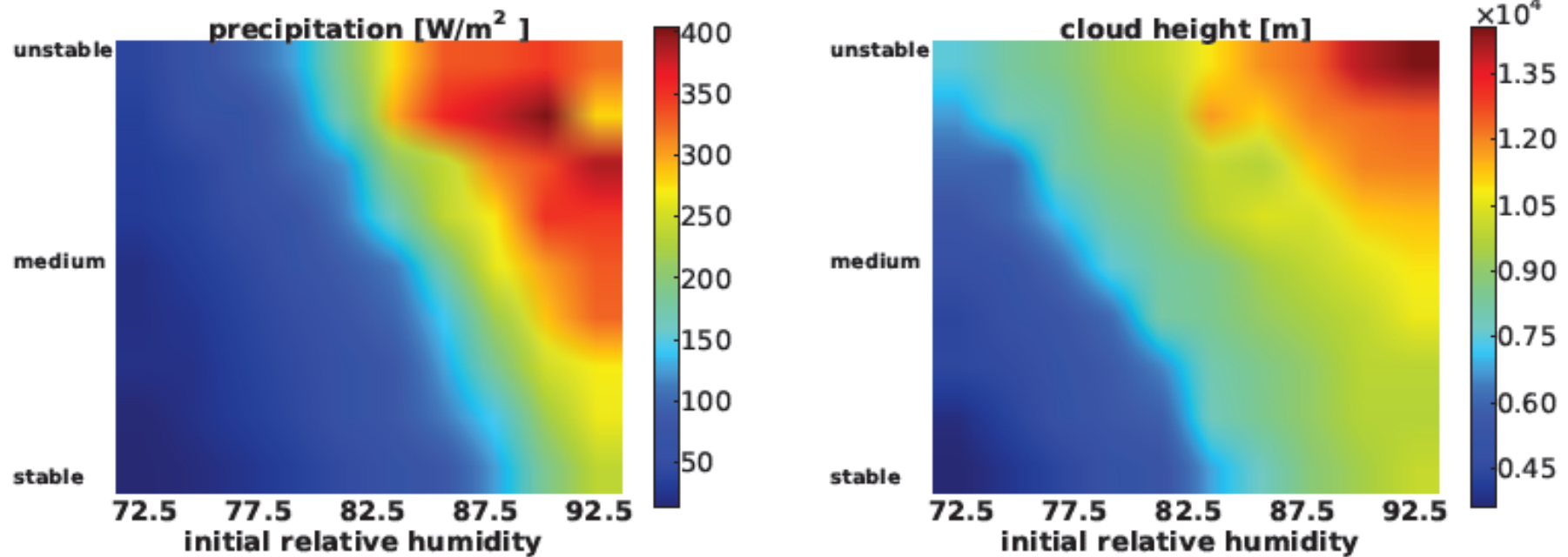
Suggesting that:



How to characterize (or parameterize) this?

precipitation and cloud top height

Cloud height $\sim 0.01 M_{\max}$

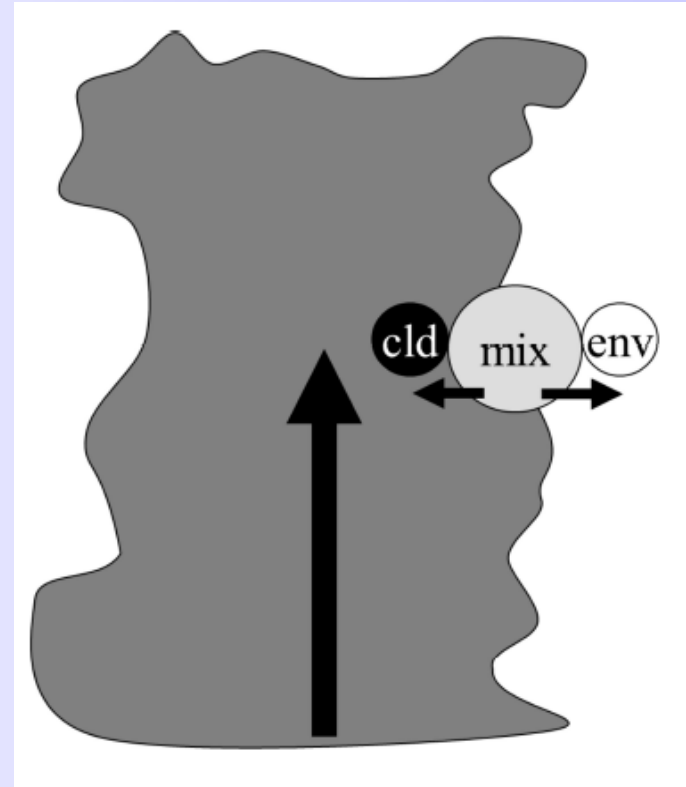


Precip , cloud top height increase with increasing RH, instability

Kain_Fritsch mixing (1) (Kain Fritsch JAS1990)

$$\frac{\partial}{\partial z} \ln M = \epsilon - \delta$$

- Fractional inflow rate ϵ_0
- Assume uniform distribution of all possible mixtures
(Bretherton et al. MWR 2004,
Raymond & Blyth JAS 86)
- Entrainment/Detrainment rate dependent on buoyancy



All positive buoyant parcels stay in the updraft (~entrainment)

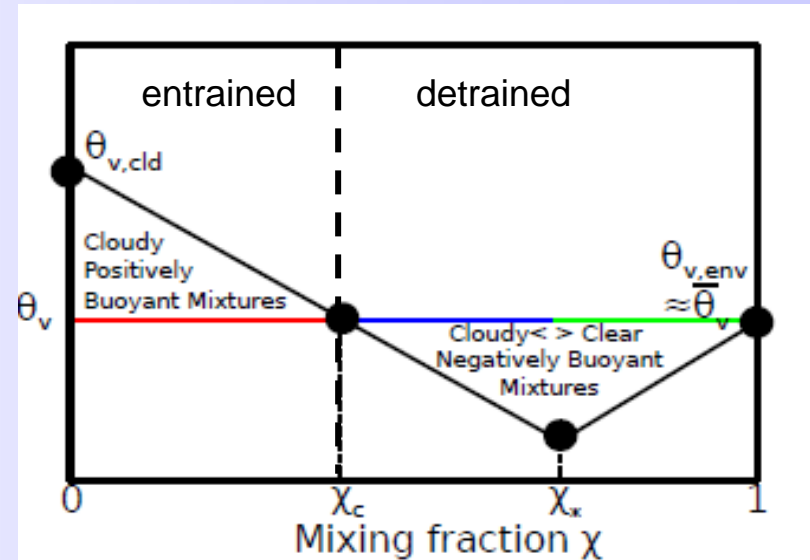
All negative buoyant parcels leave the updraft (~detrainment)

Kain_Fritsch mixing (2) (Kain Fritsch JAS1990)

$$\epsilon = 2 \int_0^{\chi_c} \chi p(\chi) d\chi = \epsilon_0 \chi_c^2$$

$$\delta = 2 \int_{\chi_c}^1 (1 - \chi) p(\chi) d\chi = \epsilon_0 (1 - \chi_c)^2$$

$$\frac{\partial}{\partial z} \ln M = \epsilon_0 (2\chi_c - 1)$$



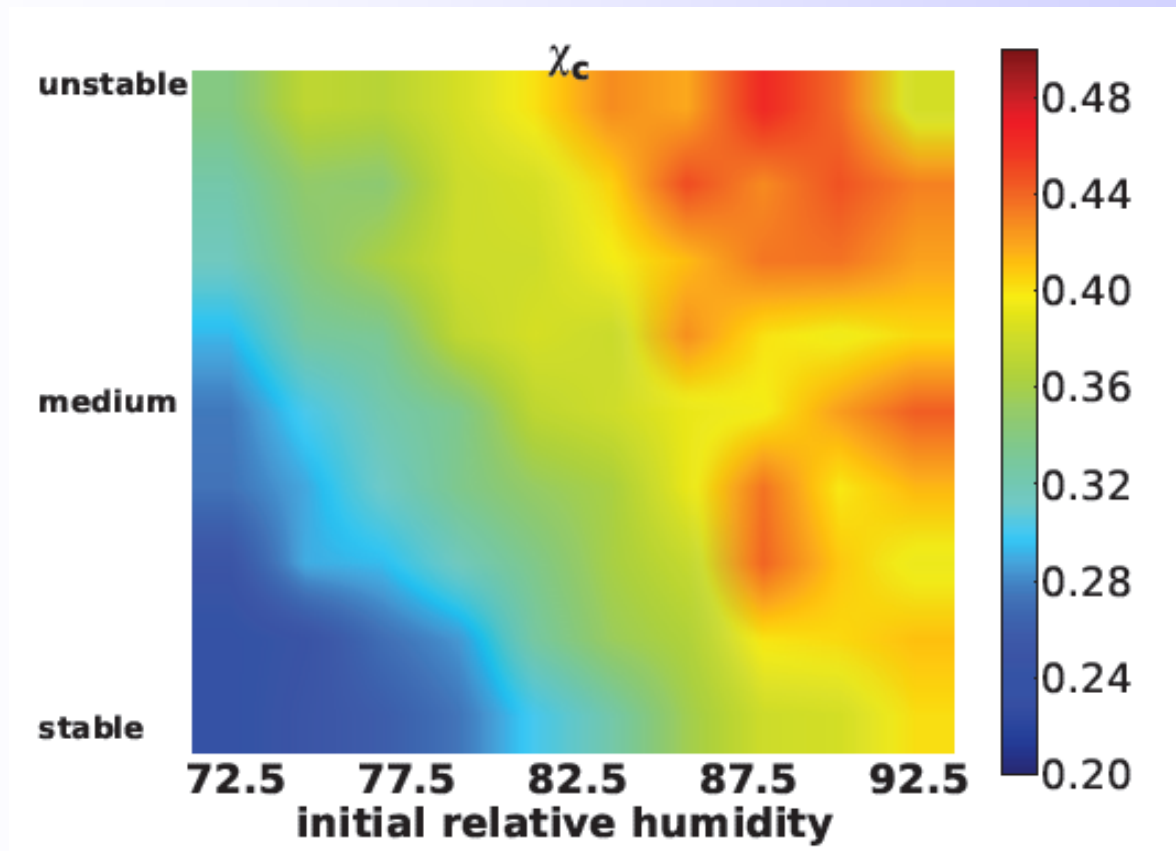
$$\chi_c = (c_p \pi / L) \frac{\Delta \theta_v}{q_{se}(\beta - \alpha)(1 - RH) - \alpha q_{lu}}$$

$$\begin{aligned} \Delta \theta_v \uparrow &=> \chi_c \uparrow \\ RH \uparrow &=> \chi_c \uparrow \end{aligned}$$

De Rooy and Siebesma MWR 2008

Less detrainment for moister and more buoyant environment

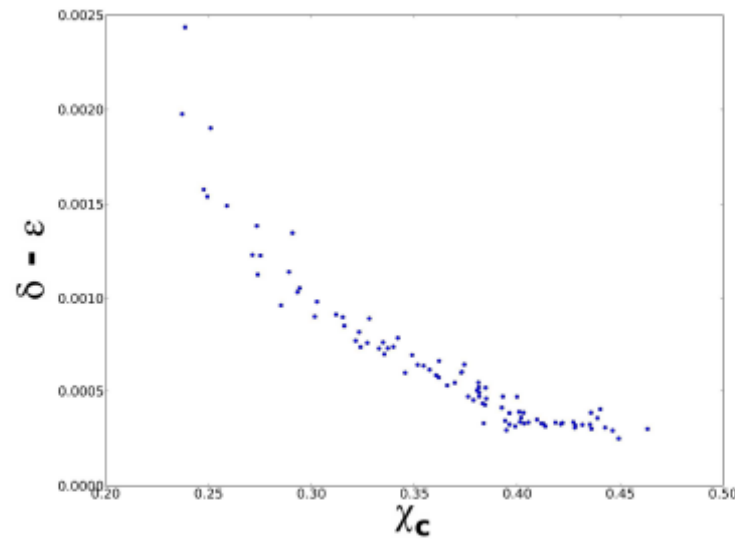
How about χ_{crit} (2~3km)?



χ_{crit} as the key parameter (2~3km)

$$\frac{\partial}{\partial z} \ln M = \epsilon - \delta$$

Strong decrease of M



Constant M

dryer and more stable

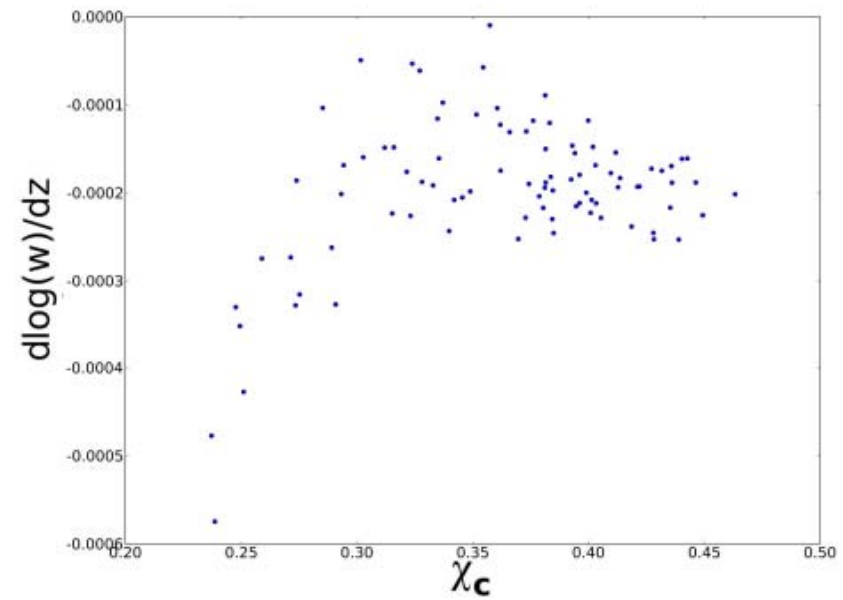
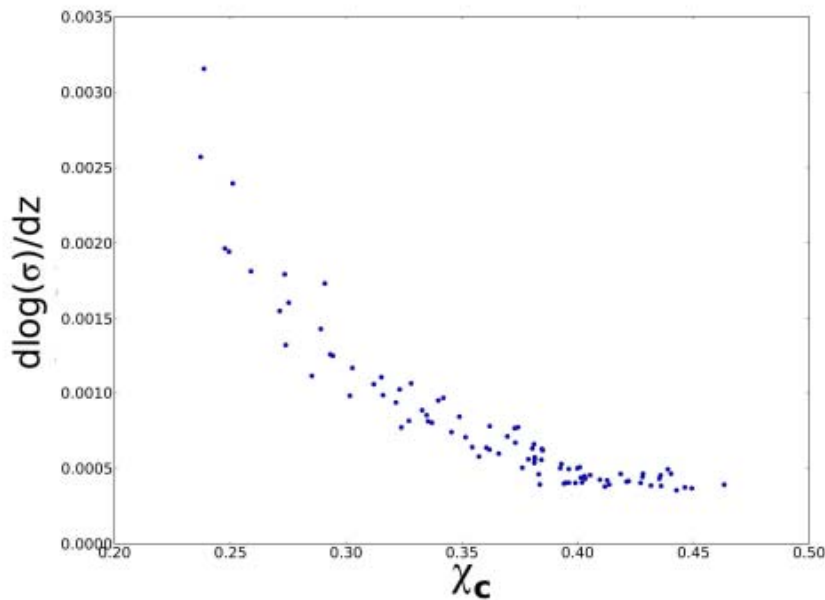
Moister and more unstable

$$M \equiv \rho_0 \sigma w_c$$

Variation due to cloud core fraction or due to incore vertical velocity?

Cloud fraction and vertical velocity

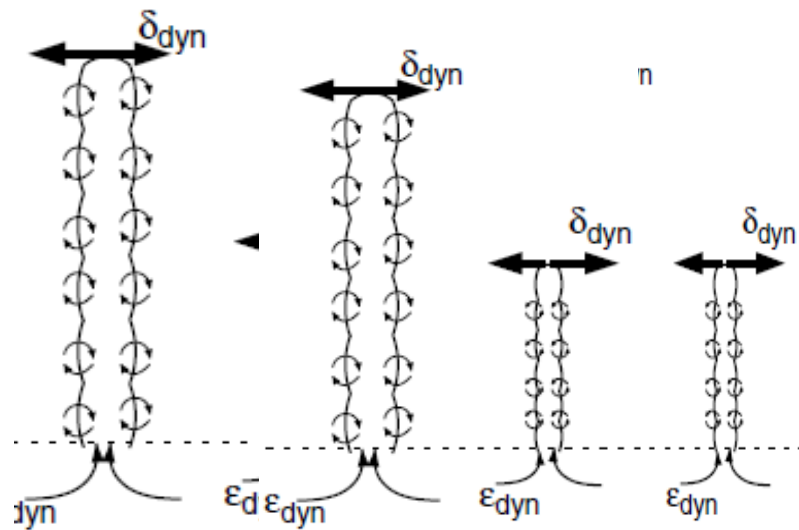
$$\frac{\partial}{\partial z} \ln M = \frac{\partial}{\partial z} \ln \sigma + \frac{\partial}{\partial z} \ln \rho_0 + \frac{\partial}{\partial z} \ln w_c$$



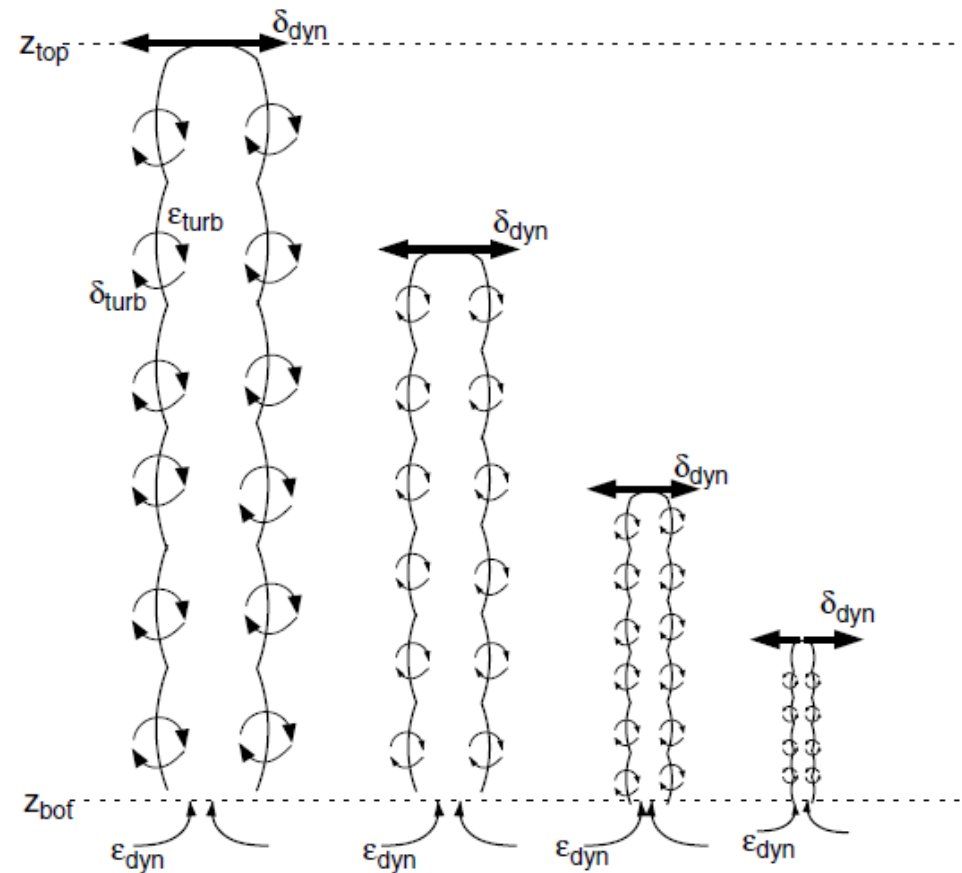
Decrease of mass flux largely due to decrease of cloud fraction

Simplified Physical Picture

Dryer and less unstable

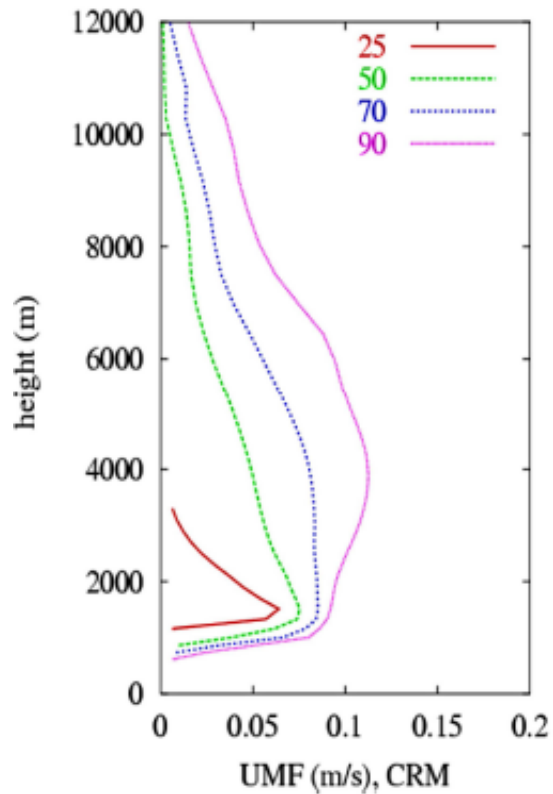


Moister and more unstable

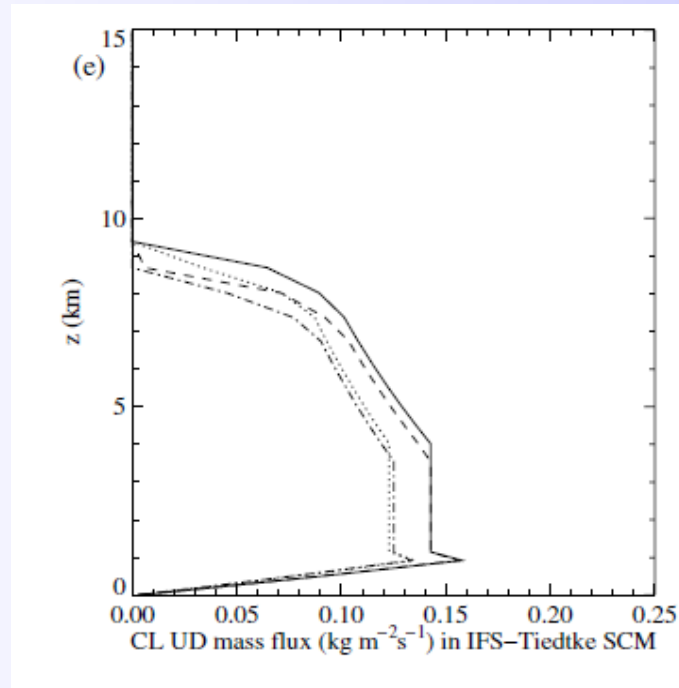


How are current convection schemes responding?

CRM



Single Column Model
(ECMWF) 2004



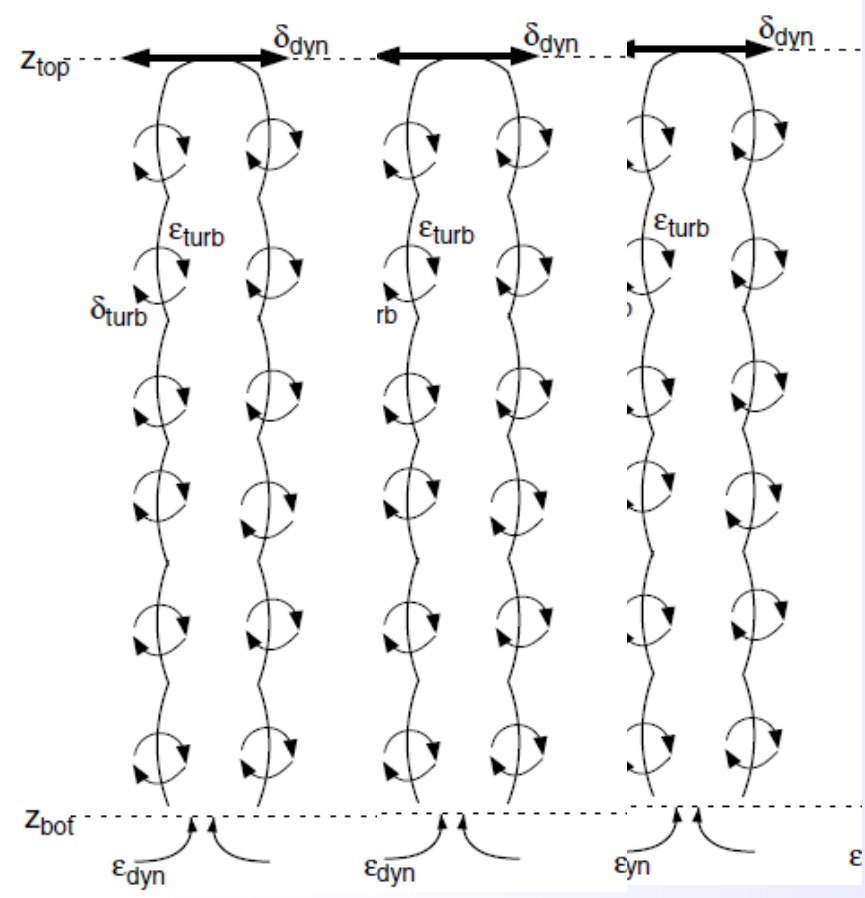
Derbyshire et al. QJRMS (2004)

Derbyshire et al. designed a case where the environmental relative humidity (RH) was varied while keeping the stability of the temperature profiles constant.

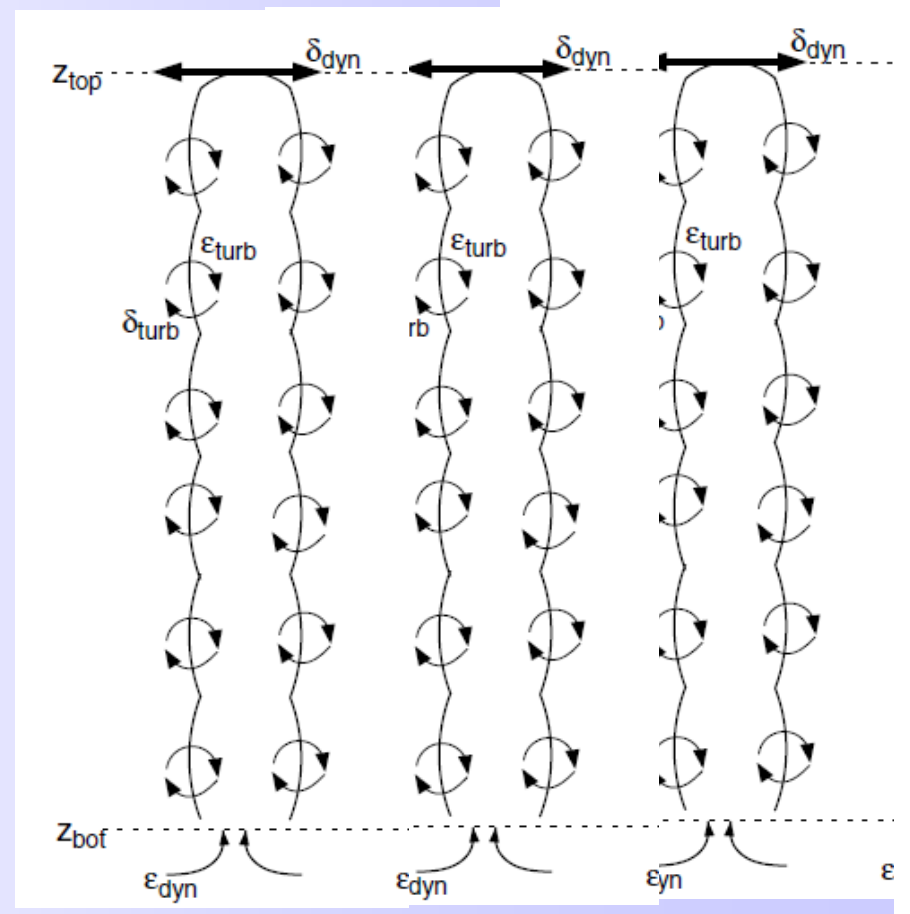
All single column models displayed convective transport insensitive to RH in clear contrast with CRM results!!! (MetO, ECMWF, ARPEGE, IPSL).

Present day Cumulus Parametrizations

Dryer



Moister



Conclusions

- Strong dependency of moist convection on tropospheric relative humidity and stability
- Mostly related to detrainment (or to the decrease of the cloud fraction)
- The critical mixing parameter χ_{crit} (Kain Fritsch 1990) seems to be a good parameter to control the detrainment (but not for entrainment!)
- Most Present day convection parameterization are too insensitive to RH.
- This is likely related to the inability of models to represent realistically the MJO.
- The Derbyshire et al 2004 study has initiated many studies around this issue:
 - (Bechtold 2008 QJRM) $\varepsilon = \varepsilon_0(1.3 - RH(z))f_{\text{scale}}$ (works well but not justified)
 - Derbyshire et al. 2011 (QJRM): adaptive detrainment
 - This present study (Boing et al. GRL2012)
- We are only beginning to constrain the convection parameterizations.
- More systematic exploration is needed
- **Do implement and test these ideas in climate models**